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CONTENTS

General Dynamical Considerations Applied to Piezoelectric Oscillations of a Crystal in an Electrical Circuit ..... W. F. G. SWANN 167

New Honors of Discovery for Dr. Joseph Priestley ..... W D. TROW 197

A System for Rapid Production of Photographic Records
F. M. BROWN, L. L. BLACKMER, AND C. J. KUNZ 203

Notes from the National Bureau of Standards 213

The Franklin Institute Notes 219

Notes from the Biochemical Research Foundation 221
Alkaline Hydrolysis of and Effect of Heat on Ribonucleic Acid
CHARLES A. ZITTLE

Book Review 228

Publications Received 228

Current Topics 196, 218, 229

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GENERAL DYNAMICAL CONSIDERATIONS APPLIED TO PIEZOELECTRIC OSCILLATIONS OF A CRYSTAL IN AN ELECTRICAL CIRCUIT.*

BY

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INTRODUCTION.

Our problem concerns the forced and free oscillations of an X-cut crystal when placed in a circuit symbolized in Fig. 1. The faces perpendicular to the x-axis form condenser plates. We shall concentrate our thoughts on the case represented by Fig. 1A, where the distance between the plates is small compared with the length and breadth of the crystal, and where the oscillations of interest are those parallel to the x-axis. However, it will be shown that, with nothing but a change in sign of the piezoelectric constant, our solutions apply equally well for the case Fig. 1B, where the length along the y-axis is large compared with the other dimensions and where the oscillations of interest are parallel to the axis of y.

In order to avoid interruption of the general run of the discussion, certain mathematical details will be relegated to an appendix and referred to as A, B, C, etc. This appendix will start, moreover, with a definition of all the relevant symbols used in the text.[1] Equations in the appendix will be numbered according to the scheme 2A, 3B, etc., and it will be convenient for reference to number some of the equations of definition in this manner.

The general solution of the problem will be developed for the case where, in addition to electrical damping as a result of electrical resistance and mechanical damping by radiation into the fluids at the end of the

* This work was done under Contract No. OEMsr-1220 under the auspices of the Office of Scientific Research and Development, which assumes no responsibility for the accuracy of the statements contained herein.

[1] The symbols will also, in general, be defined as they occur except when their meaning is obvious. In all cases, however, Appendix A serves as a final reference.

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crystal, there is also internal damping in the crystal itself. We shall confine ourselves to the case where this internal damping is determined by the rate of change of strain. However, for the purpose of reducing the solutions to certain special cases of practical interest, and in such form as to exhibit their important features, such reductions will be made for the case where this internal damping is zero. No fundamental

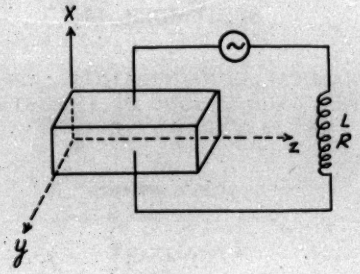

FIG. 1A.

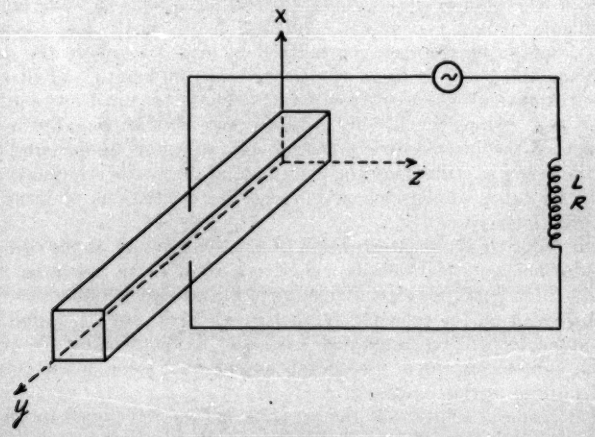

FIG. 1B.

difficulty presents itself in developing the ultimate solutions for the case where internal damping is included, however.

For the interest of those concerned with the ultimate results rather than the developments, it will be well to commence by giving a "Summary of Conclusions," which is not intended to be exhaustive, but rather to call attention to certain features which are of special interest.

GENERAL CONCLUSIONS.

1. The application of Maxwell's generalized electrodynamic procedure leads to a solution of the problem in a form in which, under a sinusoidal applied e.m.f., the crystal vibrates in general in an infinite number of Fourier modes, with the frequency of the applied e.m.f., and expressions for the amplitudes and phase angles of the oscillations of the ends of the crystal are determined, as are also the corresponding quantities for the individual modes.

2. Incidentally, it may be mentioned that, (as will be shown under the heading "A More Intuitive Approach") the fundamental dynamical equations inside the crystal are controlled entirely by the density and ordinary elasticity of the crystal. They have nothing whatever to do with the piezo-electric constant. The equations concerned with the conditions at the boundaries do, however, involve the piezo-electric constant.

An understanding of the above matters is not necessary for the development of the general dynamical theory, although it is, of course, contained implicitly in it. Such an understanding is, however, helpful in reviewing the results from an intuitive standpoint. Thus, for example, it results that the acoustical velocity in the crystal condenser has nothing whatever to do with the piezo-electric constant, but is determined entirely by the density and *ordinary* elastic constant of the material.

3. The most interesting case is that of resonance determined by the condition $p^2 - n^2k^2$, where p is 2π times the applied frequency, k is 2π times the lowest natural frequency of the crystal as determined by its elastic constant, density and length (parallel to the x-axis) and n is an integer.

It turns out that when a value n_2 satisfying the above relation is even, there is no resonance, but the crystal vibrates in an infinite number of modes with n odd, the amplitudes of the even modes being zero.

When a value n_1 satisfying the above relation is odd, the whole displacement *at the two ends* of the crystal is given by one simple harmonic term of frequency determined by n_1. The displacement is symmetrical at the two ends in spite of the difference of the radiation pressures resulting from the different media.

The single mode realized with an odd value (n_1) provides no stresses to balance the radiation pressures. In the case where the media are the same at the two ends, these radiation pressures are entirely provided for by the piezo-electric phenomenon which is symmetrical in its action for this purpose. However, when the media are different there arise an infinite number of modes with n even. These all add up to zero displacement at the two ends, but they give pressures unsymmetrical at the two ends. The amplitudes of these even modes are, in fact, pro-

portional to the difference between the $\rho_m c_m$ values of the media at the two ends, where ρ_m is the density and c_m the acoustical velocity.

4. As further elaboration of (3) above, it may be stated that for odd resonance ($p^2 = n_1^2 k^2$), and for an applied e.m.f. of the form Ve^{ipt}, the displacements ζ_l and ζ_0 at the two ends of the crystal are given by

$$\zeta_l = -\zeta_0 = \frac{-4\epsilon V e^{ipt} e^{-i\varphi}}{Ap^2\left[\left(Lp - \frac{1}{Cp}\right)^2 + \left(R + \frac{8\epsilon^2}{Ap^2 r}\right)^2\right]^{\frac{1}{2}} r}$$

where

$$\tan\varphi = \frac{(Lp - 1/Cp)}{R + 8\epsilon^2/Ap^2 r}$$

and

$$r = 2(\rho_{m_1} c_l + \rho_{m_0} c_0)$$

and where the symbols are defined in Appendix A. While the charge σ per unit area of the condenser plates is related to ζ_l by the expression

$$\sigma = q = \frac{jpr\zeta_l}{4\epsilon}.$$

5. Strictly speaking, resonance does not occur exactly at the values of p corresponding to the natural frequencies of the crystal, but at a value of p differing therefrom by an amount δp which is small. In a typical case it amounts to only 0.01 per cent, for tuning to the lowest frequency, and it diminishes for tuning to the higher harmonics.

GENERAL DYNAMICAL PROCEDURES.

The equations of Lagrange, as exemplified by the case of a typical coordinate q_r, are:

$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{q}_r}\right) - \frac{\partial K}{\partial q_r} + \frac{\partial F}{\partial \dot{q}_r} + \frac{\partial W}{\partial q_r} = \Phi_r. \qquad (1)$$

As applied to our present problem, we have one generalized electrical coordinate. It will be called q (without subscript), and is defined as $D/4\pi$, where D is the dielectric displacement. Its time rate of change is proportional to the total current i in the Maxwellian sense,[2] so that it itself is an appropriate quantity to function as an electrical coordinate of the system. The significance of the mechanical coordinates will appear later. K and F are, respectively, the kinetic energy and dissipation functions, electrical and mechanical, for the whole system. W

[2] In the wire i is wholly conductional; in the crystal it is wholly displacement current. The former characteristic, and indeed the strict validity of all that follows, is limited to the case where the *electromagnetic* wavelengths involved are large compared with the dimensions of the circuit. This assumption is involved in all analogous considerations of this problem.

is the potential energy, electrical and mechanical, of the crystal, and Φ, the generalized external force. In our case, it is wholly electrical. In general, it may be in part derivable from a potential and in part not so expressed, or it may be wholly of one or the other of these characters. In our present problem, it will be proportional to the applied e.m.f., and will be of the form $AVe^{j\nu t}$ (where j is the square root of -1), since q refers to unit area.

Expression for the Potential Energy W of the System. Piezo-electric theory makes a generalized potential energy expression the starting point for the development of the piezo-electric relations and identifies, subsequently, the constants involved with the various elastic and piezo-electric constants of the system. The end result, as applied to a problem of the kind exemplified in Fig. 1A, is a realization of relations of the form [3]

$$X = Ys - \epsilon q, \tag{2}$$

$$E_x = \frac{4\pi}{\kappa} q - \epsilon s, \tag{3}$$

where X is the stress (regarded as a tension) and E_x the field parallel to the x-axis at any point, s is the strain, Y is Young's modulus, ϵ the piezo-electric constant and κ the specific inductive capacity.[4]

In practice, students of the subject are apt to be more familiar with the relations (2) and (3) than they are with the parent w, the potential energy per unit volume; we shall consequently reverse the strictly logical procedure by deducing w from (2) and (3) through the relations

$$X = \frac{\partial w}{\partial s} \quad (4); \qquad E_x = \frac{\partial w}{\partial q}, \tag{5}$$

so that

$$\frac{\partial X}{\partial q} = \frac{\partial E_x}{\partial s}. \tag{6}$$

---

[3] See, for example, "Electro-Mechanical Transducers and Wave Filters" by W. P. Mason, Page 327. In the case represented by Fig. 1B, the relations are of exactly the same form, except that ϵ is preceded by a positive sign in both equations if ϵ is a positive quantity, and, of course, X is replaced by the y-component to correspond to stresses and strains parallel to the y-axis.

[4] Sometimes the piezo-electric relations are expressed in terms of the polarization rather than in terms of $q(=D/4\pi)$. The advantage of using q is that, since div $D = 0$, its value is independent of x, which is not true of the polarization when s varies with x in the course of the mechanical oscillations. Moreover, q is the proper coordinate in terms of which, through its time derivative, to express the electromagnetic kinetic energy.

q is, of course, equal to σ, the charge density on one of the condenser plates, but formally we must recognize $q(=D/4\pi)$ as having meaning at all points in the crystal, whereas σ has meaning only on the condenser plates.

It is easy to deduce (Appendix B) from these relations the result

$$w = \frac{Ys^2}{2} - esq + \frac{2\pi}{\kappa} q^2, \tag{7}$$

so that, if A is the area of one of the surfaces in contact with the fluid, and l the crystal length perpendicular to that area, we have, since q is independent of x,

$$W = \frac{AY}{2} \int_0^l s^2 dx - Aeq \int_0^l s dx + \frac{2\pi l A}{\kappa} q^2, \tag{8}$$

$x = 0$ and $x = l$ corresponding to the two ends of the crystal.

Expression for the Kinetic Energy K of the System. If ζ is the displacement of a particle of the medium parallel to the axis of x, the mechanical kinetic energy of the crystal per unit volume is $\rho\dot{\zeta}^2/2$. In addition to this, there is electrical kinetic energy of the system as a whole to the amount $L\dot{\imath}^2/2$. Since

$$\imath = A\frac{d\sigma}{dt} = A\dot{q}, \tag{9}$$

we have, for the electrical kinetic energy, the expression $LA^2\dot{q}^2/2$. Thus

$$K = \frac{1}{2} LA^2\dot{q}^2 + \frac{A\rho}{2} \int_0^l \dot{\zeta}^2 dx. \tag{10}$$

The Dissipation Function F. It results directly from the equations of Lagrange that, in the absence of external forces of the type Φ_r,

$$-\frac{d}{dt}(K + W) = 2F.$$

Hence, in order to find F, all we have to do is to find an expression for the rate of dissipation of energy.

The rate of dissipation of electrical energy is $R\imath^2$, so that the corresponding contribution F_e to F is $R\imath^2/2 = RA^2\dot{q}^2/2$.

It is easy to show (Appendix B) that the rate of dissipation of energy by acoustical radiation into the media symbolized by subscripts ml and mo is $A\rho_{ml}c_{ml}\dot{\zeta}_l^2 + A\rho_{mo}c_o\dot{\zeta}_o^2$, so that the corresponding contribution F_r to F is given by

$$F_r = \frac{A}{2}(\rho_l c_{ml}\dot{\zeta}_l^2 + \rho_{mo}c_o\dot{\zeta}_o^2). \tag{11}$$

The nature of the energy dissipation within the crystal itself is less well defined. Insofar as we consider it at all, we shall suppose that the

dissipation is determined by a contribution to the stress which is proportional to the time rate of change of strain. This will give rise to an energy dissipation of the form $\frac{b}{2}\left[\frac{d}{dt}\left(\frac{\partial\zeta}{\partial x}\right)\right]^2$ per unit of volume, and to a corresponding contribution F_i to F of the form

$$F_i = A\frac{b}{2}\int_0^l\left[\frac{d}{dt}\left(\frac{\partial\zeta}{\partial x}\right)\right]^2 dx. \tag{12}$$

Thus the complete expression for the dissipation function F is

$$F = \frac{A}{2}\left(\rho_{m1}c_1\dot{\zeta}_l^2 + \rho_{m0}c_0\dot{\zeta}_0^2\right) + A\frac{b}{2}\int_0^l\left[\frac{d}{dt}\left(\frac{\partial\zeta}{\partial x}\right)\right]^2 dx + \frac{RA^2\dot{q}^2}{2}. \tag{13}$$

Introduction of Mechanical Generalized Coordinates. It is possible to expand ζ in a Fourier series over the range $x = 0$ to $x = l$ in the form

$$\zeta = \frac{a_0}{2} + a_1\cos\frac{\pi x}{l} + a_2\cos\frac{2\pi x}{l} + \cdots. \tag{14}$$

In utilizing (8), (10) and (13) in (1), we shall have to evaluate the integrals contained in them. This we proceed to do as follows: Since $s \equiv \partial\zeta/\partial x$

$$s^2 = \sum_{n=1}^{\infty} a_n^2\left(\frac{n\pi}{l}\right)^2\sin^2\frac{n\pi x}{l} + \sum_{\substack{m=1\\ m\neq n}}^{\infty}\sum_{n=1}^{\infty}\left(\frac{m\pi}{l}\right)\left(\frac{n\pi}{l}\right)a_m a_n\sin\frac{m\pi x}{l}\sin\frac{n\pi x}{l}\cdots. \tag{15}$$

On integrating from $x = 0$ to $x = l$, the product terms go out, and we obtain

$$\int_0^l s^2 dx = \frac{\pi^2}{2l}\sum_1^{\infty} n^2 a_n^2. \tag{16}$$

Again,

$$\dot{\zeta} = \left(\dot{a}_1\cos\frac{\pi x}{l} + \cdots \dot{a}_n\cos\frac{n\pi x}{l} + \cdots\right) + \frac{\dot{a}_0}{2}$$

and

$$\int_0^l \dot{\zeta}^2 dx = \frac{l}{2}\sum_1^{\infty}\dot{a}_n^2 + \frac{l\dot{a}_0^2}{4}, \tag{17}$$

also

$$\int_0^l\frac{\partial\zeta}{\partial x}dx = \zeta_l - \zeta_0 = -2(a_1 + a_3 + a_5 + \cdots). \tag{18}$$

Since, in (13), the time differentiation is represented by putting a dot over ζ, and since this is simply the equivalent of dotting the a's in (14), it is clear that the integral in (13) is the same as the integral of s^2 given by (16), except that the a's are all dotted. We thus obtain, by incorporating (16), (17), and (18) in (8), (10), and (13),

$$W = \frac{AY\pi^2}{4l}\sum_1^\infty n^2 a_n^2 + 2A\epsilon q(a_1 + a_3 + a_5 + \cdots) + \frac{2\pi Al}{\kappa} q^2 \cdots, \quad (19)$$

$$K = \frac{1}{2} LA^2\dot{q}^2 + \frac{A\rho l}{4}\sum_1^\infty \dot{a}_n^2 + \frac{A\rho l \dot{a}_0^2}{8} \cdots, \quad (20)$$

$$\left.\begin{aligned} F = {} & \frac{A\rho_{m1}}{2} c_1 \left(\frac{\dot{a}_0}{2} - \dot{a}_1 + \dot{a}_2 - \dot{a}_3 + \cdots\right)^2 \\ & + \frac{A\rho_{m0}}{2} c_0 \left(\frac{\dot{a}_0}{2} + \dot{a}_1 + \dot{a}_2 + \dot{a}_3 + \cdots\right)^2 + \frac{RA^2\dot{q}^2}{2} \cdots \\ & + \frac{Ab\pi^2}{4l}\sum_1^\infty n^2 \dot{a}_n^2, \end{aligned}\right\} \quad (21)$$

where, as already stated, the symbols are defined in Appendix A.

We are now in a position to apply (1), with $\Phi_r = 0$ for the mechanical coordinates represented by the a's and $\Phi_r = AVe^{ipt}$ for the electrical coordinate represented by q. We thus obtain for the mechanical coordinates

$$\begin{aligned} & \frac{1}{2} A\rho l \ddot{a}_n + A\rho_{m1} c_1 \left(\frac{\dot{a}_0}{2} - \dot{a}_1 + \dot{a}_2 - \dot{a}_3 + \cdots\right)(-1)^n \\ & \quad + \frac{Ab\pi^2}{2l} n^2 \dot{a}_n + A\rho_{m0} c_0 \left(\frac{\dot{a}_0}{2} + \dot{a}_1 + \dot{a}_2 + \cdots\right) \\ & + \frac{AY\pi^2}{2l} n^2 a_n - A\epsilon q[-1 + (-1)^n] = 0 \cdots, \quad n = 1 \text{ to } \infty \quad (22) \end{aligned}$$

also

$$\begin{aligned} & \frac{A\rho l}{2} \ddot{a}_0 + A\rho_{m1} c_1 \left(\frac{\dot{a}_0}{2} - \dot{a}_1 + \dot{a}_2 - \dot{a}_3 + \cdots\right) \\ & \quad + A\rho_{m0} c_0 \left(\frac{\dot{a}_0}{2} + \dot{a}_1 + \dot{a}_2 + \dot{a}_3 + \cdots\right) = 0 \quad (23) \end{aligned}$$

and for the electrical coordinate

$$LA^2\ddot{q} + RA^2\dot{q} + \frac{4\pi Alq}{\kappa} + 2A\epsilon(a_1 + a_3 + a_5 + \cdots) = AVe^{ipt} \cdots. \quad (24)$$

On introducing the definitions given in Appendix A, equations (22),

(23), and (24) become, respectively:

$$\ddot{a}_n + g_1\left(\frac{\dot{a}_0}{2} - \dot{a}_1 + \dot{a}_2 - \cdots\right)(-1)^n + g_0\left(\frac{\dot{a}_0}{2} + \dot{a}_1 + \dot{a}_2 + \cdots\right)$$
$$+ g_1 n^2 \dot{a}_n + k^2 n^2 a_n + \nu q[-1 + (-1)^n] = 0, \quad n = 1 \text{ to } \infty \tag{25}$$

$$\ddot{a}_0 + g_1\left(\frac{\dot{a}_0}{2} - \dot{a}_1 + \dot{a}_2 - \dot{a}_3 + \cdots\right)$$
$$+ g_0\left(\frac{\dot{a}_0}{2} + \dot{a}_1 + \dot{a}_2 + \cdots\right) = 0, \tag{26}$$

$$\ddot{q} + \lambda\dot{q} + hq - \mu(a_1 + a_3 + \cdots) = Ne^{i\nu t}. \tag{27}$$

From here on, we shall neglect internal friction and put $g_1 = 0$.

It results, from (25) and (26), that, *for n even*:

$$\ddot{a}_n - \ddot{a}_0 + k^2 n^2 a_n = 0 \tag{28}$$

and for *n odd*

$$\ddot{a}_n + \ddot{a}_0 + 2g_0\left(\frac{\dot{a}_0}{2} + \dot{a}_1 + \dot{a}_2 + \cdots\right) + k^2 n^2 a_n - 2\nu q = 0. \tag{29}$$

Now, from (14)

$$\zeta_1 = \frac{a_0}{2} + \sum_{\text{even}} a_n - \sum_{\text{odd}} a_n \tag{30}$$

and

$$\zeta_0 = \frac{a_0}{2} + \sum a_n, \quad n = 1 \text{ to } \infty. \tag{31}$$

Hence, from (26), (30), and (31)

$$\ddot{a}_0 + g_1\dot{\zeta}_1 + g_0\dot{\zeta}_0 = 0. \tag{32}$$

Incorporating (31) in (29) and then using (32), we find:

$$\ddot{a}_n - g_1\dot{\zeta}_1 + g_0\dot{\zeta}_0 + k^2 n^2 a_n - 2\nu q = 0 \qquad \text{for } n \text{ odd.} \tag{33}$$

Using (32) in (28), we obtain

$$\ddot{a}_n + g_1\dot{\zeta}_1 + g_0\dot{\zeta}_0 + k^2 n^2 a_n = 0 \qquad \text{for } n \text{ even.} \tag{34}$$

In the steady state, a_n is sinusoidal with frequency $p/2\pi$, so that we may write

$$\ddot{a}_n = -p^2 a_n$$

where p is real. Thus, (33) and (34) yield

$$(k^2 n^2 - p^2)a_n = 2\nu q + g_1\dot{\zeta}_1 - g_0\dot{\zeta}_0 \qquad \text{for } n \text{ odd,} \tag{35}$$

$$(k^2 n^2 - p^2)a_n = -g_1\dot{\zeta}_1 - g_0\dot{\zeta}_0 \qquad \text{for } n \text{ even.} \tag{36}$$

Using the quantities S and T defined in Appendix A, equations (1A) and (2A), equations (35) and (36) yield

$$\sum_{n \text{ odd}} a_n = S(2\nu q + g_1\zeta_1 - g_0\zeta_0), \tag{37}$$

$$\sum_{n \text{ even}} a_n = -T(g_1\zeta_1 + g_0\zeta_0). \tag{38}$$

Since $\ddot{a}_0 = -p^2 a_0$, equation (32) yields

$$g_1\zeta_1 + g_0\zeta_0 = p^2 a_0. \tag{39}$$

Then (37) and (38), when substituted in (30) and (31), yield, respectively

$$\zeta_1 = \frac{a_0}{2} - T(g_1\zeta_1 + g_0\zeta_0) - S(2\nu q + g_1\zeta_1 - g_0\zeta_0) \cdots \tag{40}$$

and

$$\zeta_0 = \frac{a_0}{2} + S(2\nu q + g_1\zeta_1 - g_0\zeta_0) - T(g_1\zeta_1 + g_0\zeta_0) \cdots. \tag{41}$$

If, in (40) and (41), we now incorporate (39), we obtain

$$g_1\left(S + T - \frac{1}{2p^2}\right)\zeta_1 - g_0\left(S - T + \frac{1}{2p^2}\right)\zeta_0 + \zeta_1 + 2\nu Sq = 0 \tag{42}$$

and

$$g_1\left(S - T + \frac{1}{2p^2}\right)\zeta_1 - g_0\left(S + T - \frac{1}{2p^2}\right)\zeta_0 - \zeta_0 + 2\nu Sq = 0. \tag{43}$$

Again, on incorporating (37) into (27), we obtain

$$\ddot{q} + \lambda\dot{q} + hq - 2\mu\nu Sq - \mu g_1 S\zeta_1 + \mu g_0 S\zeta_0 = Ne^{ipt}. \tag{44}$$

Equations (42)–(44) inclusive constitute our differential equations for the desired quantities ζ_0, ζ_1, q.

Again, introducing the definitions in Appendix A, equations (4A), we obtain from (42)–(44) inclusive

$$g_{11}\zeta_1 - g_{20}\zeta_0 + \zeta_1 + \nu_1 q = 0, \tag{45}$$

$$g_{21}\zeta_1 - g_{10}\zeta_0 - \zeta_0 + \nu_1 q = 0, \tag{46}$$

$$\ddot{q} + \lambda\dot{q} + h_1 q + g_{30}\zeta_0 - g_{31}\zeta_1 = Ne^{ipt}. \tag{47}$$

Since ζ_1, ζ_0, and q are sinusoidal with frequency $p/2\pi$, these equations yield

$$(1 + jpg_{11})\zeta_1^* - jpg_{20}\zeta_0^* + \nu_1 q^* = 0, \tag{48}$$

$$jpg_{21}\zeta_1^* - (1 + jpg_{10})\zeta_0^* + \nu_1 q^* = 0, \tag{49}$$

$$-jpg_{31}\zeta_1^* + jpg_{30}\zeta_0^* + (h_1 - p^2 + jp\lambda)q^* = N, \tag{50}$$

where $j \equiv (-1)^{\frac{1}{2}}$, and the stars are intended to indicate complex amplitudes involving, of course, the phase angles.

The solutions of these equations are

$$\zeta_1^* = \frac{\begin{vmatrix} 0 & -jpg_{20} & \nu_1 \\ 0 & -(jpg_{10}+1) & \nu_1 \\ N & jpg_{30} & (h_1 - p^2 + jp\lambda) \end{vmatrix}}{\Delta} \tag{51}$$

$$\zeta_0^* = \frac{\begin{vmatrix} 0 & \nu_1 & (1 + jpg_{11}) \\ 0 & \nu_1 & jpg_{21} \\ N & (h_1 - p^2 + jp\lambda) & -jpg_{31} \end{vmatrix}}{\Delta} \tag{52}$$

$$q^* = \frac{\begin{vmatrix} 0 & 1 + jpg_{11} & -jpg_{20} \\ 0 & jpg_{21} & -(jpg_{10}+1) \\ N & -jpg_{31} & jpg_{30} \end{vmatrix}}{\Delta} \tag{53}$$

where

$$\Delta = \begin{vmatrix} 1 + jpg_{11} & -jpg_{20} & \nu_1 \\ jpg_{21} & -(jpg_{10}+1) & \nu_1 \\ -jpg_{31} & jpg_{30} & (h_1 - p^2 + jp\lambda) \end{vmatrix}. \tag{54}$$

These equations yield [5]

$$\begin{aligned}\Delta = &- h_1 + p^2 + p^3\lambda(S+\alpha)G + 4h_1p^2g_0g_1S\alpha \\ &- 4p^4g_0g_1S\alpha + 4p^2\nu_1\mu\alpha g_0g_1S + j[p^3(S+\alpha)G - h_1p(S+\alpha)G \\ &+ 4p^3\lambda g_0g_1S\alpha - p\lambda - p\nu_1\mu SG],\end{aligned} \tag{55}$$

$$\zeta_1^* = \frac{2N\nu S[1 + 2jpg_0\alpha]}{\Delta}, \tag{56}$$

$$\zeta_0^* = \frac{-2N\nu S[1 + 2jpg_1\alpha]}{\Delta}, \tag{57}$$

$$q^* = \frac{-N[1 - 4p^2g_0g_1S\alpha + jp(S+\alpha)(g_0+g_1)]}{\Delta}, \tag{58}$$

where

$$\alpha \equiv T - \tfrac{1}{2}p^2. \tag{59}$$

[5] In deducing the relations in question, it is convenient to observe from equations (4A) of Appendix A, that

$$\left.\begin{aligned} &g_{11} + g_{10} = (S+\alpha)(g_0+g_1);\ g_{21} + g_{30} = \mu S(g_0+g_1) \\ &g_{11}g_{10} - g_{21}g_{20} = 4g_0g_1S\alpha \\ &g_{11}g_{30} - g_{21}g_{20} + g_{31}g_{10} - g_{31}g_{20} = 4\alpha\mu g_0g_1S \end{aligned}\right\} \tag{60}$$

It will be observed that $\zeta_1^* = -\zeta_0^*$ for $S = \infty$, i.e. for $\alpha = 0$

If we denote by $|\zeta_1^*|$, $|\zeta_0^*|$, $|q^*|$, the real amplitudes, we find

$$|\zeta_1^*| = \frac{2N\nu S(1 + 4p^2g_0^2\alpha^2)^{\frac{1}{2}}}{|\Delta|}, \tag{61}$$

$$|\zeta_0^*| = \frac{2N\nu S(1 + 4p^2g_1^2\alpha^2)^{\frac{1}{2}}}{|\Delta|}, \tag{62}$$

$$|q^*| = \frac{N[(1 - 4p^2g_0g_1S\alpha)^2 + p^2(S + \alpha)^2(g_0 + g_1)^2]^{\frac{1}{2}}}{|\Delta|}, \tag{63}$$

where

$$|\Delta| = \{[p^2 - h + 2\mu\nu S + p^2\lambda(S + \alpha)G - 4p^2(p^2 - h)g_0g_1S\alpha]^2 + [p(p^2 - h)(S + \alpha)G + 2\mu\nu pS\alpha G + 4p^2\lambda g_0g_1S\alpha - p\lambda]^2\}^{\frac{1}{2}}. \tag{64}$$

The Case of Approximate Resonance. If the crystal were vibrating without damping under the condition $V = 0$, and with zero piezo-electric coefficient, its natural modes would correspond to wavelengths given by $4(l/2)$, $4(l/4)$, $4(l/6)$, etc., and the corresponding frequencies would be given by $c/2l$, $2c/2l$, $3c/2l$, etc. Since $c^2 = Y/\rho$, we have the squares of 2π times the natural frequencies given by $(4\pi^2Y/4\rho l^2)n^2$, where n is an integer, i.e., by k^2n^2, since $k^2 = \pi^2Y/\rho l^2$ (see Appendix A).[6] Now it turns out that when there is an applied V, the system does not resonate exactly to the frequencies given by $p^2 = k^2n^2$, but it does so approximately (see Appendix E) for odd values of n. For even values of n there is no resonance, as we shall presently see. Thus it is of interest to consider the case of $p^2 = n^2k^2$ where n is odd. For this case, we have (see Appendix B, Problem III) $S = \infty$, $\alpha = 0$; further, $S\alpha = -\pi^2/16k^2p^2$. Thus equations (56)–(58) inclusive, after some reduction and taking into account the definitions in Appendix A, yield, for n odd

$$\zeta_1 = -\zeta_0 = \frac{-4\epsilon Ve^{ipt}e^{-i\varphi}}{Ap^2\left[\left(Lp - \frac{1}{Cp}\right)^2 + \left(R + \frac{8\epsilon^2}{Ap^2r}\right)^2\right]^{\frac{1}{2}}r}, \tag{65}$$

where $r \equiv 2(\rho_{m0}c_1 + \rho_{m0}c_0)$

$$\tan\varphi = \frac{Lp - 1/Cp}{R + 8\epsilon^2/Ap^2r}, \tag{66}$$

$$q = \frac{Ve^{ipt}e^{-j\left(\varphi + \frac{\pi}{2}\right)}}{Ap\left[\left(Lp - \frac{1}{Cp}\right)^2 + \left(R + \frac{8\epsilon^2}{Ap^2r}\right)^2\right]^{\frac{1}{2}}}. \tag{67}$$

[6] It turns out that this result is true even when the piezo-electric constant is finite, Y being still the ordinary elastic constant. (See next Section—A More Intuitive Approach.)

These equations are obviously of a form to appeal very readily to our intuitions as founded upon ordinary circuit theory. We see that, *regardless of differences in the media at the two ends of the crystal, both ends vibrate with the same amplitude.*

For n even, we have (see Appendix B) $S = 0$, $\alpha = \infty$, $S\alpha = -\pi^2/16k^2p^2$. Thus the numerators of (62) and (63) are finite, but Δ, as given by (64), is infinite for a finite G. Hence the modes with n even are not excited.

Returning to the case where n is odd, it is interesting to consider certain limiting cases.

In the case where the term $1/Cp$ in the denominators is negligible compared with the term Lp

$$q = \frac{Ve^{jpt}e^{-j\left(\varphi+\frac{\pi}{2}\right)}}{Ap\left[L^2p^2 + \left(R + \frac{8\epsilon^2}{Ap^2r}\right)^2\right]^{\frac{1}{2}}}$$

$$\zeta_1 = -\zeta_0 = \frac{-4\epsilon Ve^{jpt}e^{-j\varphi}}{Ap^2\left[L^2p^2 + \left(R + \frac{8\epsilon^2}{Ap^2r}\right)^2\right]^{\frac{1}{2}}r}.$$

Hence in this case the amplitude is independent of the length of the crystal.

In the case where the term $1/Cp$ is all important in relation to the other terms in the denominator

$$|q^*| = \frac{VC}{A}$$

$$|\zeta_1^*| = |\zeta_0^*| = \frac{2\epsilon VC}{Ap(\rho_{m1}c_1 + \rho_{m0}c_0)}.$$

Since C is inversely proportional to l, $|\zeta_1^*|$ and $|\zeta_0^*|$ are inversely proportional to the length of the crystal.

A MORE INTUITIVE APPROACH.

While the foregoing derivation is direct, general, and complete, it is of interest to consider the problem from a more intuitive standpoint for the interest of those who are accustomed to think of piezo-electric oscillations in a crystal in terms of a combination of modes, analogous to the ordinary oscillations in an organ pipe, for example. In this connection, it is of interest to consider the values of the a_n's. The evaluation of these quantities is given in Appendix C, and it will be sufficient here to summarize the results as follows:

The Nature of the a_n's. If p is such that $p^2 = n_1^2k^2$ where n_1 is an odd integer, then a_{n1} is the only one of the a_n *with odd n* which is finite. All of the a_n with even n (including $n = 0$) are finite, but they add up to zero as far as the motions of the two ends of the crystal are concerned.

If p is such that $p^2 = n_2^2 k^2$, where n_2 is even, all of the a_n with n odd are finite, and all of the a_n with n even or zero are zero.

Thus, the motions of the ends of the crystal are given by the single harmonic term determined by a_{n1}.

The Nature of the Dynamics of the Crystal. We can view matters in the following light:

1. The dynamical equation must apply at each point within the crystal.

2. The dynamical equation must apply at the boundaries of the crystal.

Now considering the one-dimensional case, with the x-axis perpendicular to the condenser plates, the dynamical equation is

$$\frac{\rho d^2\zeta}{dt^2} = \frac{\partial X}{\partial x}, \tag{68}$$

where X is the stress.

Recalling equation (2), it is observed that q is independent of x. Hence, the dynamical equation inside is exactly the same as if there were no piezo-electric phenomenon. This fact illustrates the importance of using the quantity q, whose divergence is zero where there is no electric charge, and whose x derivative in our case is therefore zero. Sometimes piezo-electric phenomena are expressed in terms of the polarization, which procedure tends to obscure the simplicity of the problem in this matter.

Thus, in view of (2), and of the fact that $s = \partial\zeta/\partial x$, our dynamical equation (68) becomes

$$\rho \frac{\partial^2\zeta}{\partial t^2} = Y \frac{\partial^2\zeta}{\partial x^2}$$

which is the ordinary wave equation with the ordinary elastic constant Y and density ρ determining a velocity of propagation $(Y/\rho)^{\frac{1}{2}}$.

It follows from all this that the *ordinary simple modes familiar in the case of a simple non-piezo-electric bar are solutions of our problem as far as the internal dynamical equation is concerned*, and we only have to combine these modes in such a way as to satisfy our boundary conditions. Here, of course, the piezo-electric constant will ultimately make its appearance.

Now with the foregoing picture in mind, let us *start* by thinking of the crystal as vibrating in simple sinusoidal fashion, as though it were an ordinary non-piezo-electric crystal, and let us suppose that it is vibrating so that $k^2 n_1^2 - p^2 = 0$ where n_1 is odd. Thus, there will be a single a viz. a_{n1}. As may be seen from (14), the displacements will be equal and opposite at the two ends, and there will be loops of displacement at those points. Under these conditions, the *internal* dynamics of the crystal will be satisfied, but the boundary forces will not be satisfied because the motion postulated corresponds to a condition of *no* stress

at the surface, whereas we must have a stress equal to $-\rho_m c_1 \dot{\zeta}_l$ at $x = l$ and $\rho_m c_0 \dot{\zeta}_0$ at $x = 0$, in order to take care of the radiation. Now this aristocratic odd mode, unwilling to supply the forces for radiation itself, invokes a number of slaves in the form of the even modes which, as shown in Appendix C, give, when taken together, no displacement at $x = 0$ or at $x = l$, but which do contribute stress there. In addition, the piezo-electric phenomenon contributes to the stress. We have, in fact, as our condition, using equation (2),

$$Ys_l - \epsilon q = -\rho_m c_1 \dot{\zeta}_l \qquad \text{at } x = l \tag{69}$$

and

$$Ys_0 - \epsilon q = \rho_m c_0 \dot{\zeta}_0 \qquad \text{at } x = 0, \tag{70}$$

where s_l and s_0 are the strains given by the even modes. We cannot get s_0 and s_l by simply differentiating the Fourier Series—equation (14)—at $x = 0$ and $x = l$, because the series suffers a discontinuity there. However, we can evaluate $s = d\zeta/dx$ infinitely near these points. This is done in Appendix D. Using the values of the a_n for n even and for the case of odd resonance, as given in Appendix C, equation (20C), we find that in the crystal, near $x = 0$ and $x = l$,

$$s_0 = \frac{d\zeta}{dx} = \frac{-A\pi^2}{4k^2 l}; \qquad s_l = \frac{A\pi^2}{4k^2 l}, \tag{71}$$

where

$$A \equiv \frac{-2Nv(g_l - g_0)pje^{jvt}}{[2\mu v/G + p^2\lambda + jp(p^2 - h)]G}. \tag{72}$$

We shall continue to use A for this long quantity.

It now remains to test whether (69) and (70) above are satisfied by a solution in the form sketched above with the a's for the case of odd resonance given by (18C), (19C), (20C), and (21C) of Appendix C, with the even terms (including $n = 0$) adding up to give zero displacement at $x = 0$ and $x = l$, as shown also in Appendix C, equation (25C). If the problem is so satisfied, we have a picture of just how the crystal succeeds in vibrating in such a way that both ends have the same amplitude. This picture involves, of course, as already stated, a single mode with $n(=n_1)$ odd, and an infinite number of modes with n even, the latter contributing nothing to the stresses. The requirement that the even modes give as a whole no displacement at $x = 0$ and $x = l$ is provided for by the forms of the a_n for n even, as is shown in Appendix C, equation (25C).

Collecting our material for substitution in (67) and (70), we then have, from (71), and using the definitions in Appendix A:

$$Ys_l = \frac{AY\pi^2}{4k^2 l} = \frac{A\rho l}{4}; \qquad Ys_0 = \frac{-A\rho l}{4}.$$

From (8C) of Appendix C, using (72) and the definitions in Appendix A,

we have

$$\epsilon q = \frac{\epsilon GA}{2(g_1 - g_0)\nu} = -\frac{\rho l(g_0 + g_1)A}{4(g_1 - g_0)}.$$

Using (7C) of Appendix C, using (72), and the definitions in Appendix A, we have

$$\rho_{m1}c_1\zeta_1 = \frac{-\rho_{m1}c_1A}{g_1 - g_0} = \frac{-\rho l g_1 A}{2(g_1 - g_0)}$$

$$\rho_{m0}c_0\zeta_0 = \frac{\rho l g_0 A}{2(g_1 - g_0)}.$$

Substituting these results in (69) and (70), and cancelling the A's and a factor ρl throughout, we conclude that these equations are true if

$$\frac{1}{4} + \frac{g_0 + g_1}{4(g_1 - g_0)} = \frac{g_1}{2(g_1 - g_0)} \tag{73}$$

and

$$-\frac{1}{4} + \frac{g_0 + g_1}{4(g_1 - g_0)} = \frac{g_0}{2(g_1 - g_0)}, \tag{74}$$

i.e., if

$$(g_1 - g_0) + g_0 + g_1 = 2g_1$$
$$(-g_1 + g_0) + g_0 + g_1 = 2g_0.$$

These equations are obviously true, so that (69) and (70) are satisfied by our solutions.

To summarize, therefore, we may say as follows: In the light of the results in Appendix C, the solutions which have been obtained in the preceding section of the paper are equivalent to the following statement: Let there be an applied e.m.f. V, and let us define N as V/AL as we have done before, where A is, of course, the crystal area.

Then the solution of our problem for odd resonance ($n = n_1$) is as follows:

1. $$\zeta_0 = -\zeta_1 = a_{n1} = \frac{-2N\nu e^{ipt}}{[2\mu\nu/G + p^2\lambda + jp(p^2 - \lambda)]G}.$$

2. In addition there are an infinite number of even terms given by

$$a_{n\,(\text{even})} = \frac{-2N\nu(g_1 - g_0)pje^{ipt}}{[2\mu\nu/G + p^2\lambda + jp(p^2 - \lambda)]G(n^2k^2 - p^2)}.$$

including $n = 0$

The even terms add up to zero at the two ends of the crystal, so that they do not affect the amplitudes, which are equal at the two ends, However, they introduce a strain unsymmetrical at the two ends, and this takes care of the inequality of the radiation pressures. Note that if $g_1 = g_0$, i.e. if the media are the same at the two ends, we have *no* even terms, and the piezo-electric stress, which provides symmetrical forces on the medium at the two ends, takes care of the radiation pressures.

THE FREE OSCILLATIONS.

If in (47) we put $N = 0$, and then in (45), (46) and (47) write

$$\zeta_l = \zeta_l^* e^{\omega t}; \qquad \zeta_0 = \zeta_0^* e^{\omega t}; \qquad q = q^* e^{\omega t}, \tag{75}$$

where ω is complex, we obtain

$$(g_{11}\omega + 1)\zeta_l^* - g_{20}\omega\zeta_0^* + \nu_1 q_0^* = 0, \tag{76}$$

$$-(g_{10}\omega + 1)\zeta_0^* + g_{21}\omega\zeta_l^* + \nu_1 q_0^* = 0, \tag{77}$$

$$-g_{21}\omega\zeta_l^* + g_{20}\omega\zeta_0^* + (\omega^2 + \lambda\omega + h_1)q_0^* = 0, \tag{78}$$

in which it is to be observed that insofar as T and S are involved in the g's and in ν_1, these quantities T and S are to be supposed given by 1A and 2A, in Appendix A, with p replaced by the complex quantity ω. Of course, the real parts of the ω's, which will be negative for meaningful cases, will determine the damping coefficients, while the imaginary parts will determine 2π times the frequencies.

Equations (76), (77) and (78) demand that

$$\begin{vmatrix} (g_{11}\omega + 1) & -g_{20}\omega & \nu_1 \\ g_{21}\omega & (g_{10}\omega + 1) & \nu_1 \\ -g_{21}\omega & g_{20}\omega & (\omega^2 + \lambda\omega + h_1) \end{vmatrix} = 0. \tag{79}$$

This equation is, of course, of infinite degree in ω, since S and T occur implicitly. On substituting any of the possible (complex) ω as in (76), (77), (78), we have three homogeneous equations, which serve to determine two amplitude ratios such as ζ_l^*/q_0^*, ζ_0^*/q_0^*, since any one of the amplitudes may be chosen arbitrarily.

In general these amplitudes will be complex, and in writing them in this form, $\zeta_l^* = |\zeta_l^*|e^{i\varphi}$ for example, the phase angle φ becomes determined.

APPENDIX A.

Definitions of Symbols.

Definitions

$Y \equiv$ Young's modulus.
$\zeta \equiv$ displacement of a particle of the medium parallel to x-axis.
$\zeta^* \equiv$ the complex amplitude of ζ.
$a_n \equiv$ coefficient of $\cos n\pi x/l$ in the expansion $\zeta = \frac{a_0}{2} + \sum_1^\infty \cos n\pi x/l$.
$a_0 \equiv$ twice the first term in the above expansion of ζ.
$s \equiv$ strain $= \partial\zeta/\partial x$.
$X \equiv$ stress.
$\rho \equiv$ density of crystal.
$\rho_{ml} \equiv$ density of medium at $x = l$.
$\rho_{m0} \equiv$ density of medium at $x = 0$.
$c \equiv (Y/\rho)^{\frac{1}{2}} =$ velocity of sound in crystal.
$c_l \equiv$ velocity of sound in medium at $x = l$.

$c_0 \equiv$ velocity of sound in medium at $x = 0$.
$\kappa \equiv$ dielectric constant.
$\epsilon \equiv$ piezo-electric constant.
$E \equiv$ electric field.
$D \equiv$ dielectric displacement.
$\sigma \equiv$ charge per unit area of plate.
$q \equiv D/4\pi = \sigma$.
$q^* \equiv$ the complex amplitude of q.
$i \equiv$ current in the electrical circuit.
$K \equiv$ kinetic energy.
$w \equiv$ potential energy per unit volume.
$W \equiv$ total potential energy.
$F \equiv$ dissipation function.
$L \equiv$ self-inductance of circuit.
$l \equiv$ length of crystal parallel to axis of x.
$A \equiv$ area of surface perpendicular to axis of x.
$R \equiv$ resistance of circuit.
$b \equiv$ coefficient of s in the expression for internal friction.
$p \equiv 2\pi$ times the applied frequency.
$\omega \equiv$ a complex natural frequency (with external potential difference absent).
$C \equiv$ capacity of crystal condenser.
$V \equiv$ amplitude of the potential difference between plates.

$$N \equiv \frac{V}{LA}.$$

$$S \equiv \sum_1^\infty \frac{1}{n_1^2 k^2 - p^2} \text{ with } n_1 \text{ odd.} \tag{1A}$$

$$T \equiv \sum_2^\infty \frac{1}{n_2^2 k^2 - p^2} \text{ with } n_2 \text{ even.} \tag{2A}$$

$$\alpha \equiv T - \tfrac{1}{2}p^2. \tag{3A}$$

$$\left.\begin{aligned}
&k^2 \equiv \frac{V\pi^2}{\rho l^2}; \quad g_1 \equiv \frac{2\rho_{m1}c_1}{\rho l}; \quad g_0 \equiv \frac{2\rho_{m0}c_0}{\rho l}; \quad h \equiv \frac{4\pi l}{A\kappa L} = \frac{1}{LC};\\
&\lambda \equiv \frac{R}{L}; \quad \mu \equiv \frac{-2\epsilon}{AL}; \quad \nu \equiv \frac{-2\epsilon}{\rho l}; \quad h_1 \equiv h - 2\mu\nu S; \quad \nu_1 \equiv 2\nu S;\\
&g_{11} \equiv g_1\left(S + T - \frac{1}{2p^2}\right); \quad g_{10} \equiv g_0\left(S + T - \frac{1}{2p^2}\right);\\
&g_{21} \equiv g_1\left(S - T + \frac{1}{2p^2}\right); \quad g_{20} \equiv g_0\left(S - T + \frac{1}{2p^2}\right);\\
&g_{31} \equiv \mu g_1 S; \quad g_{30} \equiv \mu g_0 S; \quad G \equiv g_0 + g_1; \quad g_i \equiv \frac{b\pi^2 n^2}{\rho l^2};\\
&r \equiv 2(\rho_{m1}c_1 + \rho_{m0}c_0).
\end{aligned}\right\} \tag{4A}$$

APPENDIX B.

Problem I. Expression for the Potential Energy per Unit Volume.

From (2) and (4)

$$w = \frac{Ys^2}{2} - esq + f_1(q), \tag{1B}$$

where $f_1(q)$ is an arbitrary function of q.

From (3) and (5)

$$w = f_2(s) - esq + \frac{4\pi}{2\kappa} q^2, \tag{2B}$$

where $f_2(s)$ is an arbitrary function of s.

By comparison of (1B) and (2B), we find

$$f_2(s) = \frac{Ys^2}{2}; \qquad f_1(q) = \frac{2\pi}{\kappa} q^2.$$

Hence

$$w = \frac{Ys^2}{2} - esq + \frac{2\pi}{\kappa} q^2. \tag{3B}$$

Problem II. Verification of Equation (11).

If ζ_{ml} refers to a displacement in the fluid medium near $x = l$, we have, for the emission of waves into the medium (without reflection) near $x = l$

$$\zeta_{ml} = f(x - c_l t). \tag{4B}$$

The force exerted by the crystal on the medium on account of the waves radiated is PA, where

$$P = - Y_l \frac{\partial \zeta_{ml}}{\partial x}, \tag{5B}$$

where Y_l is the bulk-elasticity modulus for the medium. Thus, using (4B), (5B) yields, at $x = l$,

$$PA = \frac{AY_l}{c_l} \dot{\zeta}_l = A\rho_{ml} c_l \dot{\zeta}_l.$$

The rate of dissipation of energy on this account is consequently $A\rho_{ml} c_l \dot{\zeta}_l^2$. A similar expression applies for the other end of the crystal, so that the two contributions to the dissipation function amount to F_r, where

$$F_r = \frac{A}{2} (\rho_{ml} c_l \dot{\zeta}_l^2 + \rho_{m0} c_0 \dot{\zeta}_0^2). \tag{6B}$$

Problem III. The Relationship between T and S and Allied Relationships.

The summations expressed in T and S can be expressed in closed trigonometric form, and T can be expressed in terms of S by a simple

formula.[7] The demonstration is as follows:

$$\cot z = \frac{1}{z} + 2z \sum_1^\infty \frac{1}{z^2 - n^2\pi^2}$$

$$\sum_1^\infty \frac{1}{\frac{n^2\pi^2}{z^2} - 1} = \frac{1}{2} - \frac{z}{2}\cot z.$$

Hence

$$\sum_1^\infty \frac{1}{\frac{k^2n^2}{p^2} - 1} = \frac{1}{2} - \frac{\pi p}{2k}\cot\frac{\pi p}{k}.$$

Now

$$p^2T = \sum_{n \text{ even}} \frac{1}{\frac{k^2n^2}{p^2} - 1} = \frac{1}{\frac{k^2}{p^2}2^2 - 1} + \frac{1}{\frac{k^2}{p^2}2^2\cdot 2^2 - 1} + \frac{1}{\frac{k^2}{p^2}2^2\cdot 3^2 - 1}$$

$$+ \frac{1}{\frac{k^2}{p^2}2^2\cdot 4^2 - 1} + \cdots = \sum_1^\infty \frac{1}{\left(\frac{2k}{p}\right)^2 n^2 - 1} = \frac{1}{2} - \frac{\pi p}{4k}\cot\frac{\pi p}{2k} \quad (7B)$$

and

$$p^2(T + S) = \sum_1^\infty \frac{1}{\frac{k^2n^2}{p^2} - 1} = \frac{1}{2} - \frac{\pi p}{2k}\cot\frac{\pi p}{k}. \quad (8B)$$

Hence

$$p^2S = \frac{\pi p}{2k}\left[\frac{1}{2}\cot\frac{\pi p}{2k} - \cot\frac{\pi p}{k}\right]. \quad (9B)$$

Put $\pi p/2k = \theta$. Then, since

$$\cot 2\theta = \frac{\cot^2\theta - 1}{2\cot\theta} = \frac{1}{2}\cot\theta - \frac{1}{2}\cdot\frac{1}{\cot\theta}$$

$$S = \frac{\pi}{4pk}\frac{1}{\cot\theta} = \left(\frac{1}{p^2}\right)\left(\frac{\pi p}{4k}\right)^2 \frac{1}{\frac{\pi p}{4k}\cot\frac{\pi p}{2k}} = \frac{1}{p^2}\left(\frac{\pi p}{4k}\right)^2 \frac{1}{\frac{1}{2} - p^2T}. \quad (10B)$$

Hence

$$T = \frac{1}{2p^2} - \frac{\pi^2}{16k^2p^2S}. \quad (11B)$$

If $\alpha \equiv T - \frac{1}{2p^2}$, we have

$$\alpha = T - \frac{1}{2p^2} = \frac{-\pi^2}{16k^2p^2S}. \quad (12B)$$

[7] The first of these matters was pointed out to me by Mr. J. F. Marshall, and the second by Dr. H. M. Schwartz.

Hence

$$\alpha S = \frac{-\pi^2}{16k^2p^2}. \tag{13B}$$

If n is odd ($=n_1$) and if $p^2 = n_1^2k^2$, S is infinite. Thus

$$T - \frac{1}{2p^2} = \alpha = 0 \quad \text{for } p^2 = n_1^2k^2, \quad \text{with } n_1 \text{ odd.} \tag{14B}$$

If n is even ($=n_2$) and if $p^2 = n_2^2k^2$, T is infinite, and $\alpha = \infty$, so that, from (12B) $S = 0$. Thus

$$\alpha = \infty, \quad T = \infty, \quad \text{and} \quad S = 0, \quad \text{if } p^2 = n_2^2k^2, \quad \text{with } n_2 \text{ even.} \tag{15B}$$

APPENDIX C.

The Determination of the a_n's for the Cases in Which p is Adjusted to the Value $p^2 = n^2k^2$ where n is an Odd or Even Integer.

On recognizing that $\ddot{a}_0 = -p^2a_0$ for the steady state, (32) yields

$$-p^2a_0 + g_1\xi_1 + g_0\xi_0 = 0. \tag{1C}$$

Then (1C), (35), and (36) yield

$$a_0 = \frac{g_1\xi_1 + g_0\xi_0}{p^2}, \tag{2C}$$

$$a_n = \frac{2\nu q + g_1\xi_1 - g_0\xi_0}{k^2n^2 - p^2} \qquad \text{for } n \text{ odd,} \tag{3C}$$

$$a_n = \frac{-g_1\xi_1 - g_0\xi_0}{k^2n^2 - p^2} \qquad \text{for } n \text{ even.} \tag{4C}$$

The values of ξ_1, ξ_0, and q are obtained by attaching the factor e^{jpt} to the expressions (56)–(58) inclusive where Δ is given by (55), and after substituting for h_1 and ν_1, the values given in Appendix A, is

$$\Delta = -h + p^2 + 2\mu\nu S + p^2\lambda(S + \alpha)G + 4p^2(h - p^2)g_0g_1S\alpha$$
$$+j[p(p^2 - h)(S + \alpha)G + 2\mu\nu pGS\alpha + 4p^3\lambda g_0g_1S\alpha - p\lambda]. \tag{5C}$$

We shall denote by n_1 an odd integer, and by n_2 an even integer, and we shall discuss the case where p is such that $p^2 = n_1^2k^2$ and the case where $p^2 = n_2^2k^2$. It will be observed that there is sense to asking for the value of a_n, with n even, for the case $p^2 = n_1^2k^2$, or for the case $p^2 = n_2^2k^2$, and there is sense to asking for the value of a_n, with n odd, for either of these two conditions.

Case where $p^2 = n_1^2k^2$, n_1 being odd. Here we have (see Appendix B), $S = \infty$, $\alpha = 0$, $\alpha S = -\pi^2/16k^2p^2$. Thus we may retain in Δ, and in the numerators of (56)–(58) only those terms which involve S in the

form other than $S\alpha$. We thus obtain

$$\Delta = [2\mu\nu/G + p^2\lambda + jp(p^2 - h)]GS \tag{6C}$$

and

$$\zeta_1 = -\zeta_0 = \frac{2NS\nu e^{ipt}}{\Delta}, \tag{7C}$$

$$q = \frac{-jpNSe^{ipt}G}{\Delta}. \tag{8C}$$

Thus, from (3C), *and for n odd*,

$$a_n = \frac{-2jpN\nu[1 - (g_1 + g_0)/G]e^{ipt}}{(k^2n^2 - p^2)[2\mu\nu/G + p^2\lambda + jp(p^2 - h)]} = 0 \quad \text{if } k^2n^2 \neq p^2. \tag{9C}$$

When $k^2n^2 = p^2$, it is necessary to proceed by a limiting process which will, in fact, amount to retaining the terms $2jpg_0\alpha$, $2jpg_1\alpha$, and $1 - 4p^2g_0g_1S\alpha$ in the numerators of (56), (57), and (58) respectively. The term $jp\alpha G$, of course, contributes nothing to (58) compared with these terms, when $\alpha = 0$, so that, remembering the cancellation of the other terms as exemplified in (9C), we obtain, after a little reduction,

$$a_n = \frac{-2\nu Ne^{ipt}}{[k^2n^2 - p^2][2\mu\nu/G + p^2\lambda - jp(p^2 - h)]GS}. \tag{10C}$$

Now, referring to S as defined in Appendix A, it is obvious that the limit of $(k^2n^2 - p^2)S$, when $k^2n^2 - p^2 = 0$, is unity. Thus, substituting the value of Δ we have

$$a_{n1} = \frac{-2\nu Ne^{ipt}}{[2\mu\nu/G + p^2\lambda + jp(p^2 - \lambda)]G}. \tag{11C}$$

For a_0, and for the case $p^2 = n_1^2k^2$, we have, from (2C), (56) and (57),

$$a_0 = \frac{2N\nu(g_1 - g_0)je^{ipt}}{[2\mu\nu/G + p^2\lambda + jp(p^2 - h)]pG}. \tag{12C}$$

For n even and not equal to zero, and for $p^2 = n_1^2k^2$, we have, from (4C), (56), and (57),

$$a_n = \frac{-2N\nu(g_1 - g_0)pje^{ipt}}{[2\mu\nu/G + p^2\lambda + jp(p^2 - h)][G(n^2k^2 - p^2)]}. \tag{13C}$$

Case where $p^2 = n_2^2k^2$, n_2 being even. Here we have (see Appendix B), $S = 0$, $\alpha = \infty$, $\alpha S = -\pi^2/16k^2p^2$. We may thus retain in Δ as given by (5C) only the terms involving α and not involving $S\alpha$.

Thus

$$\Delta = [p^2\lambda + jp(p^2 - h)]\alpha G. \tag{14C}$$

The only finite terms in the numerators of (56) and (57) are those involving α. Also the dominant term in the numerator of (58) is

$-jp\alpha GN$. Thus

$$\zeta_1 = \frac{4N\nu p g_0 S j e^{ipt}}{[p^2\lambda - jp(p^2 - h)]G} \tag{15C}$$

$$\zeta_0 = \frac{-4N\nu p g_1 S j e^{ipt}}{[p^2\lambda - jp(p^2 - h)]G}$$

$$q = \frac{-Np j e^{ipt}}{[p^2\lambda + jp(p^2 - h)]}.$$

Thus, for n odd, (3C) yields, after observing that $S = 0$,

$$a_n = \frac{-2N\nu p j e^{ipt}}{[k^2n^2 - p^2][p^2\lambda + jp(p^2 - h)]}. \tag{16C}$$

In view of (15C), $g_1\zeta_1 + g_0\zeta_0 = 0$, so that, from (2C),

$$a_0 = 0.$$

Also, from (4C)

$$a_n = 0 \qquad \text{if } k^2n^2 \neq p^2.$$

If $k^2n^2 = p^2$, we must return to (56) and (57), and proceed by a limiting process, using, however, the value of Δ given by (14C). Thus, using (4C),

$$a_n = \frac{2N\nu Sp(g_0 - g_1)je^{ipt}}{[k^2n^2 - p^2][p^2\lambda + jp(p^2 - h)]\alpha G}.$$

Now, from Appendix B

$$\alpha(k^2n^2 - p^2) = T(k^2n^2 - p^2) - \frac{k^2n^2 - p^2}{2p^2}.$$

When n^2 approaches the limit $n_1^2 (= p^2/k^2)$, the second term on the right hand side becomes zero and $T(k^2n^2 - p^2)$ approaches unity (see Appendix B). Thus, since we still have S in the numerator of a_n, we find

$$a_{n1} = 0. \tag{17C}$$

Collection of Results. We then have as follows: If p is adjusted to the value given by $p^2 = n_1^2k^2$, where n_1 *is an odd integer*,

$$a_{n1} = \frac{-2\nu Ne^{ipt}}{[2\mu\nu/G + p^2\lambda + jp(p^2 - h)]G} \qquad \text{and is finite,} \tag{18C}$$

$$a_n = 0 \qquad \text{if } n \text{ is odd and not equal to } n_1, \tag{19C}$$

$$a_n = \frac{-2N\nu(g_1 - g_0)pje^{ipt}}{[2\mu\nu/G + p^2\lambda + jp(p^2 - h)][G(n^2k^2 - p^2)]}, \tag{20C}$$

if n is even and not zero. It is finite.

$$a_0 = \frac{2N\nu(g_1 - g_0)je^{ipt}}{[2\mu\nu/G + p^2\lambda + jp(p^2 - h)]pG} \qquad \text{and is finite.} \tag{21C}$$

If p is adjusted to the value given by $p^2 = n_2^2 k^2$ where n_2 *is an even integer*,

$$a_n = \frac{-2Nvpje^{ipt}}{[p^2\lambda + jp(p^2 - h)][k^2n^2 - p^2]}, \tag{22C}$$

if n is odd. It is finite.

$$a_n = 0 \qquad \text{for all } n \text{ (including } n_2\text{) if } n \text{ is even,} \tag{23C}$$

$$a_0 = 0. \tag{24C}$$

To show that at $x = 0$ *or* $x = l$

$$\frac{a_0}{2} + \sum_2^\infty a_n \cos\frac{n\pi x}{l} = 0$$

where n is even, and where the a's have the values given (for $p^2 = n_1^2k^2$, *with* n_1 *odd) by (20C) and (21C).*

From the equations (20C) and (21C) above referred to, it is clear that

$$a_n = \frac{-p^2a_0}{n^2k^2 - p^2} \qquad \text{for } n \text{ even and } > 0.$$

Thus, at $x = 0$ or $x = l$, since n is even,

$$\frac{a_0}{2} + \sum_2^\infty a_n \cos\frac{n\pi x}{l} = \frac{a_0}{2} + \sum_2^\infty a_n$$

$$= \frac{a_0}{2} - p^2a_0 \sum_2^\infty \frac{1}{n^2k^2 - p^2} \qquad n \text{ even}$$

$$= \frac{a_0}{2} - p^2Ta_0.$$

However, as will be seen from Appendix B, equation (14B), $T = \frac{1}{2p^2}$ for the case in question. Hence, at $x = 0$ or $x = l$,

$$\frac{a_0}{2} + \sum_2^\infty a_n \cos\frac{n\pi x}{l} = 0. \tag{25C}$$

APPENDIX D.

Evaluation of $\frac{d\zeta_e}{dx}$ near $x = 0$ and $x = l$ from the Series $\zeta_e = \frac{a_0}{2} + a_2\cos\frac{2\pi x}{l} + a_4\cos\frac{4\pi x}{l} + \cdots$ etc. where ζ_e is the Displacement Given by the Even Terms of (14), and where the a's are Given for the Case $p^2 = n_1^2k^2$, with n_1 Odd, by (12C) and (13C) in Appendix C.

Except at $x = 0$ and $x = l$, we have

$$\frac{d\zeta_e}{dx} = -\sum_2^\infty \frac{n\pi a_n}{l} \sin\frac{n\pi x}{l}$$

n even

Write [8]

$$K \equiv \sum_{2}^{\infty} \frac{n\pi a_n}{l} \sin \frac{n\pi x}{l}$$

where

$$a_n = \frac{A}{n^2k^2 - p^2}$$

with

$$A = \frac{-2N\nu(g_1 - g_0)pje^{ipt}}{[2\mu\nu/G + p^2\lambda + jp(p^2 - h)]G}$$

as may be seen from (13C) of Appendix C. Thus

$$K = \frac{\pi A}{l} \sum_{2}^{\infty} \frac{n}{n^2k^2 - p^2} \sin \frac{n\pi x}{l} = \frac{\pi A}{lk^2} \sum_{2}^{\infty} \frac{n^2k^2 - p^2 + p^2}{n(n^2k^2 - p^2)} \sin \frac{n\pi x}{l}$$

$$K = \frac{\pi A}{lk^2} \sum_{2}^{\infty} \frac{1}{n} \sin \frac{n\pi x}{l} + \frac{p^2\pi A}{lk^2} \sum_{2}^{\infty} \frac{1}{n(n^2k^2 - p^2)} \sin \frac{n\pi x}{l}$$

for n even

Let us consider a large value n_0 of n, and a value of x so near to 0 or l that the modulus of $\sin \frac{n_0\pi x}{l}$ is less than a small quantity σ. Then

$$\sum_{2}^{\infty} \frac{1}{n(n^2k^2 - p^2)} \sin \frac{n\pi x}{l} = \sum_{2}^{n_0-1} \frac{1}{n(n^2k^2 - p^2)} \sin \frac{n\pi x}{l} + \sum_{n_0}^{\infty} \frac{1}{n(n^2k^2 - p^2)} \sin \frac{n\pi x}{l}$$

for n even

$$\text{i.e.} < \sigma \sum_{2}^{\infty} \frac{1}{n(n^2k^2 - p^2)} + \sum_{n_0}^{\infty} \frac{1}{n(n^2k^2 - p^2)}$$

$$\text{i.e.} < \sigma \sum_{2}^{\infty} \frac{1}{n^2k^2 - p^2} + \frac{1}{n_0} \sum_{n_0}^{\infty} \frac{1}{n^2k^2 - p^2}$$

$$\text{i.e.} < \sigma \sum_{2}^{\infty} \frac{1}{n^2k^2 - p^2} + \frac{1}{n_0} \sum_{2}^{\infty} \frac{1}{n^2k^2 - p^2}$$

for n even

$$\text{i.e.} < \left(\sigma + \frac{1}{n_0}\right) T,$$

where T is defined in Appendix A. For the case $p^2 = n_1^2k^2$, with n_1 odd, we have $T = 1/2p^2$ (see Appendix B, equation (14B)).

[8] We here use the symbol K and later σ, temporarily, for purposes other than those defined in Appendix A.

Hence, since σ is zero in the limit, and $1/n_0$ is zero in the limit

$$\sum_{\substack{2 \\ n \text{ even}}}^{\infty} \frac{1}{n(n^2k^2 - p^2)} \sin \frac{n\pi x}{l} = 0.$$

Thus

$$K = \frac{A\pi}{k^2 l}\left[\frac{1}{2}\sin\frac{2\pi x}{l} + \frac{1}{4}\sin\frac{4\pi x}{l} + \cdots \text{etc.}\right].$$

Put $2x = y$. Then

$$K = \frac{A\pi}{2k^2 l}\left[\sin\frac{\pi y}{l} + \frac{1}{2}\sin\frac{2\pi y}{l} + \frac{1}{3}\sin\frac{3\pi y}{l} + \cdots\right]$$

$$= \frac{A\pi}{2k^2 l}\left[\sin\frac{\pi y}{l} + \frac{1}{3}\sin\frac{3\pi y}{l} + \frac{1}{5}\sin\frac{5\pi y}{l} + \cdots\right] \qquad y = 0 \text{ to } 2l$$

$$+ \frac{A\pi}{2k^2 l}\left[\frac{1}{2}\sin\frac{2\pi y}{l} + \frac{1}{4}\sin\frac{4\pi y}{l} + \cdots\right].$$

Now near $x = 0$, y is near $y = 0$, and consequently, if K_0 refers to the value of K near to $x = 0$,

$$K_0 = \frac{A\pi}{2k^2 l}\left[\sin\frac{\pi y}{l} + \frac{1}{3}\sin\frac{3\pi y}{l} + \frac{1}{5}\sin\frac{5\pi y}{l} + \cdots\right] + \frac{1}{2}K_0.$$

Now (see, for example, Byerly's "An Elementary Treatise on Fourier's Series and Spherical, Cylindrical, and Ellipsoidal Harmonics," page 51, example 1) the series in the last equation is $\pi/4$, from $y = 0$ to $y = l$. Hence

$$K_0 = \frac{A\pi^2}{8k^2 l} + \frac{1}{2}K_0.$$

Hence

$$K_0 = \frac{A\pi^2}{4k^2 l}.$$

Now if n is even, $\sin\frac{2\pi nx}{l}$ has the same value for all values of n, for x a little greater than zero, as it has for x greater than l by the same amount. Thus, if K_l is the value of K for x a little less than l

$$K_l = -K_0.$$

Thus

$$\frac{d\zeta_e}{dx} = \frac{-A\pi^2}{4k^2 l} \text{ for } x \text{ just greater than zero and}$$

$$\frac{d\zeta_e}{dx} = \frac{A\pi^2}{4k^2 l} \text{ for } x \text{ a little less than } l.$$

APPENDIX E.

Approximation to True Resonance.

So far we have investigated, as a special case, the oscillations for the case $p^2 = n^2k^2$, where n is odd. This corresponds to cases where the applied frequency is equal to one of the natural frequencies of the crystal. While a general experience of vibrating systems suggests that the condition $p^2 = n^2k^2$ gives a frequency very near to that for which the amplitudes have maxima, we have not actually shown this to be the case. The following treatment establishes this point formally. We shall discuss the amplitude $|\zeta_1^*|$. The case for $|\zeta_0^*|$ will follow in analogous manner. From (61) and (64), we have

$$\frac{|\zeta_1^*|^2}{(2N\nu)^2} = \frac{S^2(1+4p^2g_0^2\alpha^2)}{\left[p^2-h+\frac{\pi^2}{4k^2}(p^2-h)g_0g_1+p^2\lambda G\alpha+(p^2\lambda G+2\mu\nu)S\right]^2+B} \tag{1E}$$

$$\text{where } B=\left[\frac{-\pi^2\mu\nu G}{8k^2p}-\frac{\pi^2pg_0g_1\lambda}{4k^2}-p\lambda+p(p^2-h)G\alpha+Gp(p^2-h)S\right]^2.$$

We shall introduce the dimensionless quantity x, given by

$$x = \alpha p^2, \tag{2E}$$

so that, from (13B),

$$\frac{1}{S} = \frac{-16k^2x}{\pi^2}. \tag{3E}$$

For $p^2 = n^2k^2$, $S = \infty$ and $x = 0$, so that x is a measure of the departure from this condition.

Substituting from (2E) and (3E) in (1E) we obtain, after a little reduction,

$$\frac{|\zeta_1^*|^2}{(2N\nu)^2}(p^2\lambda G+2\mu\nu)^2 = \frac{1+\left(\frac{2g_0}{p}\right)^2x^2}{\left[1-\frac{\left\{\frac{p^2}{\lambda G}\left(1-\frac{h}{p^2}\right)\left(1+\frac{\pi^2g_0g_1}{4k^2}\right)+x\right\}\frac{16k^2}{p^2}\frac{x}{\pi^2}}{1+\frac{2\mu\nu}{\lambda Gp^2}}\right]^2+\frac{p^2\left(1-\frac{h}{p^2}\right)^2}{\lambda^2\left(1+\frac{2\mu\nu}{p^2\lambda G}\right)^2}H} \tag{4E}$$

$$\text{where } H=\left[1+\frac{\left\{\frac{\pi^2\mu\nu}{8p^2k^2}+\frac{\pi^2g_0g_1\lambda}{4k^2G}+\frac{\lambda}{G}-\left(1-\frac{h}{p^2}\right)x\right\}\frac{16k^2}{p^2\pi^2}x}{1-\frac{h}{p^2}}\right]^2.$$

Now put

$$\left.\begin{aligned}
&\alpha'=\left(\frac{2g_0}{p}\right)^2 && \lambda'=1+\frac{2\mu\nu}{\lambda G p^2} && \epsilon'=\frac{\pi^2\mu\nu}{8p^2k^2}+\frac{\pi^2 g_0 g_i}{4k^2 G}+\frac{\lambda}{G}\\
&\beta'=\left(\frac{p^2}{\lambda G}\right)\left(1-\frac{h}{p^2}\right) && \mu'=\frac{16k^2}{p^2\pi^2}\Big/1+\frac{2\mu\nu}{\lambda G p^2} && \omega'=\frac{16k^2}{p^2\pi^2}\Big/1-\frac{h}{p^2}\\
&\gamma'=1+\frac{\pi^2 g_0 g_i}{4k^2} && \nu'=\frac{p^2}{\lambda^2}\frac{\left(1-\frac{h}{p^2}\right)^2}{\left(1+\frac{2\mu\nu}{p^2\lambda G}\right)^2} && \rho'=\left(1-\frac{h}{p^2}\right).
\end{aligned}\right\} \quad (5E)$$

Writing X for the left hand side of (4E), we have

$$X=\frac{1+\alpha' x^2}{[1-(\beta'\gamma'+x)\mu' x]^2+\nu'[1+(\epsilon'-\rho' x)\omega' x]^2}$$

$$=\frac{1+\alpha' x^2}{1+\nu'+2(\epsilon'\omega'\nu'-\beta'\gamma'\mu')x+(\beta'^2\gamma'^2\mu'^2-2\mu'-2\rho'\omega' x^2+\epsilon'^2\omega'^2\nu')x^2}. \quad (6E)$$

Now while the quantities other than x in this expression depend upon p, they do not have the characteristic which x has of being a very rapidly varying function of p (see (2E)) in the neighborhood of $x=0$. Hence it is legitimate to find from (5E) the value of x which makes X a maximum, treating all the quantities other than x as constants. On evaluating $\frac{dX}{dx}$ and equating to zero, neglecting [9] squares and higher powers of x, we obtain

$$x=\frac{\epsilon'\omega'\nu'-\beta'\gamma'\mu'}{(1+\nu')\alpha'-\beta'^2\gamma'^2\mu'^2+2\mu'+2\rho'\omega'\nu'-\epsilon'^2\omega'^2\nu'}. \quad (7E)$$

Now it is possible to show [10] that

$$-n\frac{\delta p}{k}=\frac{8}{\pi^2}x. \quad (8E)$$

---

[9] A procedure justified by the ultimate value of x obtained.

[10] From (3E) and (10B)

$$\frac{-16k^2}{\pi^2}x=\frac{1}{S}=\frac{4pk}{\pi}\cot\frac{\pi p}{2k}.$$

Putting $\pi p/2k=\theta$, we have

$$\frac{-16xk}{\pi^2 p}=\frac{4}{\pi}\frac{\cos\theta}{\sin\theta}=\frac{4}{\pi}\frac{\sin\left(\frac{\pi}{2}-\theta\right)}{\cos\left(\frac{\pi}{2}-\theta\right)}=\frac{4}{\pi}\frac{\sin\frac{\pi}{2}\left(1-\frac{p}{k}\right)}{\cos\frac{\pi}{2}\left(1-\frac{p}{k}\right)}.$$

Put $p=nk+\delta p$. Then, neglecting second order quantities,

$$-\frac{4x}{\pi}=n\frac{\sin\frac{\pi}{2}\left[(n-1)+\frac{\delta p}{k}\right]}{\cos\frac{\pi}{2}\left[(n-1)+\frac{\delta p}{k}\right]}.$$

When $\delta p/k$ is small and n is odd this becomes

$$-n\frac{\delta p}{k}=\frac{8}{\pi^2}x.$$

We shall now take the following values for the constants applicable to a specific case, all quantities being in c.g.s. units or in c.g.s. electrostatic units.

$$\left.\begin{array}{llll} \rho=2.65; & \rho_{ml}=1; & \rho_{m0}=0.001; & \epsilon=1.43\times10^{5}; \\ A=1; & l=0.01; & c_l=1.4\times10^{5}; & c_0=3.3\times10^{4}; \\ L=1.1\times10^{-17}; & R=5.5\times10^{-11}; & C=37; & c=5.65\times10^{5}. \end{array}\right\} \quad (9E)$$

This corresponds to a quartz crystal vibrating with one plate in contact with water and the other in contact with air, the fundamental frequency $k/2\pi$ being 2.8×10^7 oscillations per second.

Substituting these values to obtain the constants in (1E), using the definitions in Appendix A, and then substituting in the set of quantities defined by (5E), we obtain, from (7E), for $p/k = 1$,

$$x = \frac{722 - 661}{0 - 436000 + 4 + 1921 - 410}$$

where we have written in the numbers to correspond to the symbols in (7E). Thus

$$x = -0.00014$$

and from (8E)

$$\frac{\delta p}{k} = 0.00010 \quad (10E)$$

where δp represents the amount by which the value of p for true resonance differs from k.

It is possible to express the quantities α', β', etc., of (5E) in such a manner as to cause them to exhibit their dependence on p/k, and to retain these forms in (7E), so that we may investigate how $\delta p/k$ varies with n when the relation $n^2k^2 - p^2 = 0$ is satisfied for n odd. It becomes possible to show that the value of $\delta p/k$ decreases with increase of n. Thus the error in tuning represented by (10E) represents the worst case.

The arguments for ζ_0 are in every way analogous to those for ζ_1, and the conclusions are essentially the same.

Pneumatic Fatigue Testing Machine.—A pneumatically operated fatigue testing machine, whose operating principle is very similar to that of a slide trombone, was described by its developer, F. B. Quinlan of the Schenectady Works Laboratory of the General Electric Company, at the recent Buffalo meeting of the American Society for Testing Materials.

Originally developed to test gas turbine buckets, the device has proved so efficient and adaptable that it will undoubtedly have wide peacetime applications, Mr. Quinlan said.

The operating mechanism of the new instrument is extremely simple, consisting of a tuned air column in which the tuning is accomplished by decreasing the length of the air path, much as a trombone player changes the tone of his instrument by moving the slide.

The part to be tested is placed in such a position between the two air paths or tubes that the air from the open ends strikes on the upper end of the sample piece, setting it to vibrating. Since the piece vibrates at the same frequency as the note in the testing trombone, and air is continually fed through the tubes, the air impulses will cause continuous vibration of the test piece. Adjusting the tube length so that the air in them vibrates at the same frequency as that of the test sample makes it possible to produce very large and stable amplitudes of vibrations with comparatively small amounts of air.

The sample vibrates, of course, until it fatigues to the extent that its vibrations are out of phase with those of the tuned air and a break occurs.

The pneumatic fatigue tester has produced stresses as high as 100,000 pounds per square inch, Mr. Quinlan said in his talk, with no more air being used than that supplied by the average vacuum cleaner. The tester has no parts to wear out, as nothing moves put the part to be broken. Three different models have been built, the one described, one that has proved especially efficient for torsion testing, and one for strip testing. All of them operate on the same principle, but differ in size and arrangements for holding the piece to be tested. With the exception of the strip fatigue machine, all parts may be tested at temperatures from below zero to 1700°F.

Current research with the pneumatic tester is providing preliminary data which indicate that by using a recording frequency meter, the chart will show what appears to be a point of incipient failure. These data, of course, are extremely valuable to the designer and engineer. Brittle materials show this point of incipient failure to be very close to that of complete fracture, the author pointed out in conclusion, but in ductile materials it appears long before visible fracture becomes apparent.

R. H. O.

NEW HONORS OF DISCOVERY FOR DR. JOSEPH PRIESTLEY.

BY

W. D. TROW,

Formerly electrician with Thomas A. Edison, Inc.; chemist with the Anaconda Mining Company, Great Falls, Montana; chemist with the Wash. Steel & Ordnance Co., Washington, D. C., etc., etc.

In a recent study of the electrical investigations of Dr. Joseph Priestley (1733–1804), the noted English Unitarian Clergyman, chemist, natural philosopher, and "electrician"; commonly known as one of the independent discoverers of oxygen, it is now established with assurance: *First:* He was the first to discover in the year 1766, that charcoal and carbon will conduct electricity. *Second:* He was first to observe in the year 1766, a luminous electric spark or feeble arc-light between a charcoal point and a metal point, which he described as "*white electric fire.*" *Third:* He was the first to establish an electric spark between two charcoal or carbon points.

To his honor therefore we must now give him credit for being the first man to produce and observe the first feeble electric arc-light, in the history of mankind. An honor that is long overdue.

Previous to World War II the writer was engaged for several years in literary research, particularly in the history of chemistry and electricity. In this endeavor it was found that the tracing of original source material and data is an interesting but difficult branch of history.

More and more it is being realized that a firm foundation in practically any branch of modern knowledge is only obtainable by a thorough and accurate study of the history of that subject.

The accomplishments of the late Thomas A. Edison were possible largely because he made it a practice of "reading all he could find" on any research or invention project he had under way. And it is to be remembered here that Edison owned and maintained one of the largest private libraries in the United States. In his library was to be found an innovation not ordinarily found in libraries—between two tiers of the bookshelves was a private cot, that was one of Edison's favorite rest-sanctums as the occasion demanded.

It is not commonly known that Dr. Joseph Priestley was one of the leading "electricians" of his time. Many of us have believed that the term "electrician" is a modern term. We find, however, that the term was commonly used in the days of Franklin and Priestley, and perhaps before, as was the term "electric light."

In 1761 Priestley became a resident of Warrington, England, where he was a teacher of languages at the Dissenting Academy. Privately

he diligently pursued his studies of chemistry and electricity. In 1766 he made a trip to London where he was elected a member of the Royal Society. While in London he was introduced to, and became acquainted with, the great American statesman, natural philosopher, and "electrician," Dr. Benjamin Franklin. As a result of this acquaintance, it is said that Dr. Franklin encouraged Priestley to complete his extensive "History and Present State of Electricity," the first edition of which was then in preparation.

Dr. Joseph Priestley was an able historian, in addition to his other accomplishments. His work, the "History and Present State of Electricity," according to reliable authorities, was subsequently printed in the following editions: London, 1767 (1st ed.); London, 1769 (2nd ed.); London, 1772 (Additions to 2nd ed.); London, 1775 (3rd ed.—2 vols.); London, 1775 (4th ed.—2 vols.); London, 1794 (5th ed.—2 vols.); Paris 1771 (French, 3 vols.) and Berlin, 1772 (German).

The collation of the first edition, London, 1767, as given by the Curator of the Edgar F. Smith Memorial Collection in the History of Chemistry, University of Pennsylvania, is as follows:

"xxxi, 736 p., 5 unnumb. leaves. 8 plates. 26½ cm."

In 1767 Dr. Priestley moved to Leeds, England, where he continued his preaching and pursuit of science.

Up to about the year 1800, the year in which the great Italian philosopher and scientist Volta announced his discovery of the galvanic pile, later to become known as the "Voltaic pile" in his honor, the principal source of a supply of electricity or "electric virtue, effluvia, or fluid" as it was spoken of then, was from rubbed glass rods or tubes, or from various types of frictional electric machines. The original electric machine being the one invented by Otto Von Guericke (1602–1686), German philosopher and inventor, born in Magdeburg, the capital of the Province of Saxony, Prussia, situated on the left bank of the Elbe river, 88 miles by rail southwest of Berlin. Guericke became burgemaster of Magdeburg in the year 1646 and the machine was invented sometime between the years 1650 and 1660, the exact date being uncertain. It consisted of a ball of sulphur about the "size of an infant's head" mounted on a shaft that could be rotated. Guericke placed his hand lightly upon the rotating sulphur ball with a stroking movement and found that electricity was generated upon the ball, which could be conducted away from it by various devices. An old illustration shows a chain suspended from a horizontal bar, dangling against the rotating ball, and some authors say that he first collected the static electricity by means of a pointed stick hanging from a thread.

These early frictional electric machines supplied a high-voltage, low-amperage, electro-static current, the strength of which depended upon the size of the machine, the atmospheric conditions, and the

dexterity of the operator to a great degree. As the machines were improved and enlarged from time to time, and after the invention of the Leyden jar in 1745–1746, with the arrangement of the jars in "batteries," the "electricians" of the day found themselves supplied with sources of stronger and stronger discharges or sparks of frictional electricity that had at times a more or less continuous feature, as was found by Priestley with his charcoal discharge points.

These early "electricians" and "natural philosophers" as they were called, did an enormous amount of experimenting and investigating. Many of them were quite careful and accurate in recording the results of their work and as time goes on, with a closer study being given to their investigations, it will undoubtedly be found that in the rush of modern advance some work of value may have been overlooked.

Virtually all historians to date have given credit to Sir Humphry Davy (1778–1829), the renowned English chemist and "natural philosopher" of the Royal Institute of England, as being the first to discover, in the year 1800, a presumable feeble arc-light between two carbon points. This credit is based upon the data in a letter from Humphry Davy to Mr. Nicholson of Nicholson's Journal, London, and published therein in October 1800, in part as follows:

"September 22, 1800, Dowry Square, Hot Wells.

"Sir: The earlier experimentors on animal electricity noticed the power of well-burned charcoal to conduct the common galvanic influence" [i.e. electric current].

"I have found that this substance possesses the same properties as metallic bodies in producing the shock and spark, when made a medium of communication between the ends of the galvanic pile of Signor Volta. . . ."

At the time Davy wrote this letter he was employed as Superintendent of the Beddoes Pneumatic Institute of Clifton, near Bristol, England, which was a medical institute for the healing of sick people by the respiration of different gases.

For the source of his "galvanic influence" [electric current] at that time Davy used "an apparatus" that "was constructed for Dr. Beddoes, and never consisted of less than 110 pairs of metallic plates."

There is no question that Davy was the first to produce an electric arc-light by the use of electric current from the "voltaic battery" the credit of which is more definitely established by his work of the year 1802, which it may be well to review here.

On Feb. 16, 1801, Davy was appointed assistant Lecturer and Director of the Chemical Laboratory at the Royal Institute in London. Six weeks later he was appointed to the lectureship, and on May 31, 1802, he was appointed Professor of Chemistry. Sometime in the year 1802, probably shortly after May 31st, he performed some experiments in the Theater of the Royal Institute (Dr. John Davy, Collected Works of Sir Humphry Davy, London, 1839, Vol. II, pp. 211–212), in which he produced a much larger spark or arc-light, by the use of a current of

"galvanic electricity" from a battery consisting of "150 series of plates of copper and zinc of 4 inches square, and 50 of silver and zinc of the same size." And continuing his description, he says: "The metals were carefully cemented into four boxes of wood in regular order, after the manner adopted by Cruickshank, and the fluid made use of was water combined with about 1/100 part of its weight of nitric acid." . . . "When the circuit in the batteries was completed by means of small knobs of brass, the spark perceived was of a dazzling brightness, and in apparent diameter at least ⅛ of an inch. It was perceived only at the moment of the contact of the metals, and it was accompanied by a noise or snap." *When instead of the metals, pieces of well-burned charcoal were employed, the spark was still larger and of a vivid whiteness.*"

At a later period, in the years 1808 and 1809, Davy produced his large and continuous arc-lights with electric current from a large voltaic battery of 2,000 cells constructed for the Royal Institute.

DR. PRIESTLEY'S DISCOVERIES.

We now turn to the historic experiments of Dr. Joseph Priestley made on the 4th day of May, A.D. 1766.

Dr. Priestley, "History and Present State of Electricity," 1st Edition, London, 1767, p. 602 et seq., remarks as follows:

"These experiments on '*mephitic air*,' deceitful as they were, led, however, to a discovery, which may possibly throw some new light upon some of the most fundamental principles of electricity. [Note by W.D.T.—At that time "mephitic air" was a term given to various gases or "airs" that would not sustain animal or human life. Priestley had been making various experiments to determine whether or not it would conduct electricity.]

"Finding that I could make nothing of '*mephitic air*' itself, for I had endeavored to procure it in a variety of ways, but without success; I considered that it was from *charcoal* that I procured it in the greatest quantity, and thought I would try charcoal itself in substance. Accordingly on May 4, 1766, I tried the charcoal, in a variety of ways and states; and found it to be, what I had suspected, an excellent conductor of electricity.

"Presenting a piece of charcoal to the prime conductor, [of his frictional electric machine] together with my finger, or a piece of brass wire, I constantly observed, that the electric spark struck the charcoal before either of the other conductors, if it happened to be advanced ever so little before them. Having a very rough surface, the charcoal did not take a dense spark from the [prime] conductor, till it was made a little smooth, and brought within about half an inch; when, to all appearance, it did quite as well as any piece of metal, *there being a constant stream of dense, and white electric fire between the* [prime] *conductor and it.* I tried the charcoal in every state of heat or cold, and found no alteration of its conducting power.

"I placed a great number of pieces of charcoal, not less than twelve or twenty, of various sizes, in a circuit, and discharged a common jar [Leyden jar] through them; when, to all appearance, the discharge was as perfect, as if so many pieces of metal had been placed in the same manner. Two of the pieces, about the middle of the circuit, I placed about an inch and a half from one another; but upon the discharge, the spark passed the interval [gap] very full and strong. A piece of charcoal also made the discharge at the wire with one spark, but the report was not so loud as when the discharge was made with a piece of metal. It was observable, that a black gross smoke rose from between each of the pieces of charcoal at the moment of the discharge; but the ignition was momentary, and the fire could not be perceived on the charcoal.

"To make the experiment of the conducting power of charcoal in the most indisputable manner, I took a piece of baked wood, which I had often used for the purpose of insulation, being an excellent non-conductor and putting it into a long glass tube, I thrust it into the fire, and converted it into charcoal. In this operation, a very great quantity of gross smoke rose from it, so that, seeming to part with more of its moisture, one would have expected it would come out a better non-conductor, but, upon trial, its electric property [i.e. of being electrified] was quite gone, and it had become a very good conductor.

"The experiments above-mentioned were first made with *wood-charcoal*, of which I found pieces of very different degrees of conducting power; but the most perfect conductors I have found of this kind are some pieces of *pit charcoal*. These seem to be, in all respects, as perfect conductors as metals. They receive a strong bright spark from the prime conductor, though seldom at above an inch distance, on account of the roughness of their surface, which cannot be taken off; and in discharging a jar through them, or with them, no person can imagine any difference between them and metal, either in the colour of the electric spark, or the sound made by the explosion. When they are broken, they exhibit an appearance which much resembles that of broken steel. There is however a great variety in the electrical properties of different pieces of this kind of charcoal; and for want of proper opportunity I have not yet succeeded in ascertaining, with sufficient certainty, the circumstances, in the preparation, etc., on which this variety depends."

Priestley then tried various samples of "pit charcoal" and found therm all to have practically the same "conducting power."

He proceeds with various experiments on different "coals" and "cinders," carbonized pieces of oak "from the same plank"; "kennel coal" which he says "is remarkable for flaming much while it burns"; "coals which yield a strong sulphurous smell when they are burnt"; all to the purpose of determining their "conducting power." He seems to use the term "charcoal" as a common term for all manner of coals, coke, or charcoal. Near the close of these experiments he remarks: "In this course of experiments I found myself much at a loss for a sufficiently accurate method of acertaining the difference [i.e. degree of conductivity] of conducting substances, and I wish that electricians would endeavour to find such a measure." He adds: "One of the best that I am acquainted with, and which I applied among others on this occasion, is by the residuum of discharges, measured by Mr. Lane's electrometer." Priestley gives us an insight here of how really an accurate and careful scientific investigator he was. He proceeds: "It is well known that the worse the conductors are that form the circuit, the greater residuum will be left in a jar after a discharge; and Mr. Lane's electrometer, which measures an explosion, will likewise measure the residuum. To apply this method with accuracy, I put pieces of charcoal, etc., of the same length into the circuit, I used the same chain in every experiment, and the same disposition of every part of the apparatus; I also made the explosions exactly equal, and after every discharge compleated the circuit by the chain before I took the residuum; and lastly, I was careful to take up the same time in each operation, which I repeated very often. This method of measuring the conducting power of substances I learned from Mr. Lane."

Priestley found, much to his surprise, that "*soot* whether of wood coal or pit coal hardly conducted at all," and "the snuff of a candle would not conduct a shock, though it was placed in the middle of the circuit, and it was easily set on fire by the explosion of a small jar."

Dr. Joseph Priestley was a young man thirty-three years of age, when he made these experiments and observations, and in youthful confidence he completes this investigation with the following remarks:

"But notwithstanding my want of success, I make no doubt, but that any person of tolerable sagacity, who has an opportunity of making experiments in a laboratory, where he could reduce to a coal all kinds of substances, in every variety of method, might very soon ascertain what it is that makes charcoal a conductor of electricity. In all the methods in which I could make charcoal, the fume of the bodies was suffered to escape; but let trials be made of substances, reduced to a coal without any communication with the open air, or where the vapours emitted from them shall meet with different degrees of resistance to their escape, ascertained by actual pressure.

"Charcoal, besides its property of conducting electricity, is, on many other accounts, a very remarkable substance; being indestructible by any method, besides burning in the open air; and yet it seems not to have been sufficiently studied by any chemist. A proper examination of it promises very fair, not only to ascertain the cause of its conducting, and, perhaps, of all conducting powers; but to be an opening to various other important discoveries in chemistry and Natural Philosophy; and the subject seems to be fairly within our reach."

"Pit Coal, and probably all other substances, at the same time that they lose much of their weight, increase considerably in their bulk in the operation of charring. Does it not seem to follow from hence that its conducting power may possibly be owing to the largeness of its pores, agreeable to the hypothesis of Dr. Franklin, that electric substances have exceeding small pores, which dispose them to break with a polish.

"Or, since the calces [oxides] of metals, which are electric bodies, become metals, and conductors, by being fused in contact with charcoal; [an observation he credits to Becarria], are not metals themselves conductors of electricity, in consequence of something they get from the charcoal?

"This course of experiments, however, evidently overturns one of the earliest, and, hitherto, universally received maxims in electricity, viz., that *water* and *metals* are conductors, and all other bodies non-conductors: for we have here a substance, which is clearly neither water, nor metal, and yet a good conductor."

It is thus established that Dr. Joseph Priestley, at thirty-three years of age, was not only a great scientific discoverer, but was also an accurate "electrician", as well.

A SYSTEM FOR RAPID PRODUCTION OF PHOTOGRAPHIC RECORDS.

BY

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Communication No. 1062 from the Kodak Research Laboratories.

ABSTRACT.

In the land-based radar direction of aircraft it was found necessary to transfer to a large plotting screen a 10× magnification of the PPI cathode-ray tube face. An instrument was developed to do this photographically, with the projected image lagging only 15 seconds behind the latest information. A single 360-degree sweep on the PPI tube is reduced to a ¼-inch diameter image on Eastman Type 5302 Fine Grain Release Positive Film. The resultant image is rapid-processed by the application of hot solutions in a special processing cup which restricts the liquids to a small circular area on the film. The spent solutions are quickly removed by a vacuum line and the near dry image is indexed into the projection system where an air pressure gate completes the drying, cools the film, and holds it flat during projection. The projected image, at a magnification of over 300×, has sufficient quality to resolve all the pertinent detail of the tube face. The method is felt to have some general application.

The system that we are about to describe was developed at the request of the Radiation Laboratory and the U. S. Army Air Forces for a specific wartime purpose. Though the specific need for this instrument has happily disappeared, we feel that the apparatus and photographic technique used in the development have other and more general applications sufficient to make this description of interest.

The military purpose for which the equipment was made may be stated briefly, as follows:

The technique of aircraft direction from land-based radar stations as worked out by the Allied Air Forces required a large-scale display showing all aircraft positions within the range of the radar system. This amounted essentially to a magnification of about ten times of the plan position indicator (PPI) cathode-ray tube with which most recent radar sets are equipped. The magnification has been accomplished in a somewhat cumbersome fashion by hand-plotting aircraft positions on a vertical edge-illuminated transparent plastic screen. In general, each plotter was given plots (in range and azimuth) by telephone from a corresponding observer at an individual cathode-ray tube. The number of plotter-observer teams and indicators varied with the amount of air traffic. Effective use of all data from radar systems having an azimuth resolution of 1 degree and ranges of 100 miles or more, for example, required ten or more indicators and teams during periods of only medium traffic densities. Speed and accuracy quite naturally suffered as traffic increased or as crew training lapsed.

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† Formerly of the Radiation Laboratory.

The unavoidable inaccuracies involved in the human linkage between indicator and screen, the large amount of equipment as well as the number of personnel required for the process, and the long training period of crews, made desirable some improvement in the method.

One of the best and least complicated methods of achieving a large magnification of the PPI is through the medium of photography. The Kodak Research Laboratories, with the technical assistance of the Radiation Laboratory, worked out the details and equipment of a

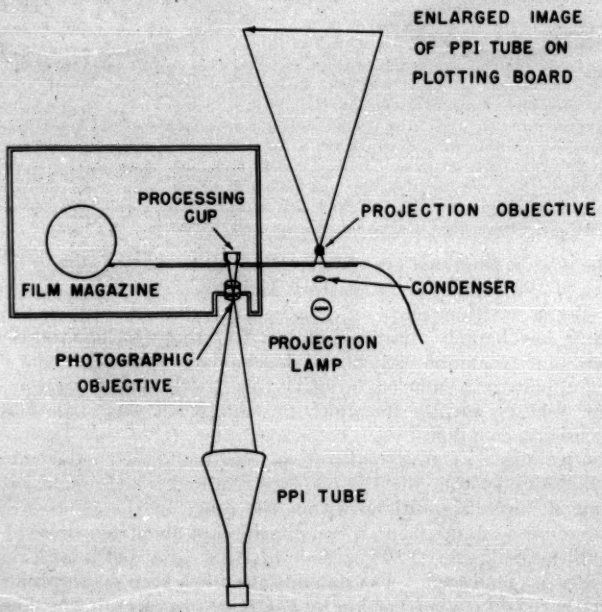

FIG. 1. Schematic drawing of P[4]I operation.

completely automatic photographic process. The required presentation is accomplished by photographing a single PPI display during one rotation of the radar antenna, processing the film in a matter of seconds, and projecting the negative image. The device is known to the services as the "Plan Position Photographic Projector" or, for brevity, P[4]I.

The P[4]I using 16-mm. film becomes, in effect, a combined camera, processing machine, and projector. The arrangement by which these three functions are carried on in a continuous automatic process is indicated schematically in Fig. 1. The continuous strip of film is

pulled intermittently from left to right, emulsion side uppermost, along a track inside a light-tight housing. During the stationary period the image is produced and a previously formed image is projected. The exposure is made through the film base and is accomplished by focusing the cathode-ray tube face through an aperture in the track. Immediately upon completion of exposure, the image, which is a circle inscribed in a 16-mm. frame, is developed, fixed, washed, and dried. Following the processing cycle, which may be as short as five seconds, the film is pulled one and one-half inches (five 16-mm. frames) to the right. This amount of movement is necessary to place the newly processed image in a position where it can be immediately projected. The four frames intervening between the image frames are not used.

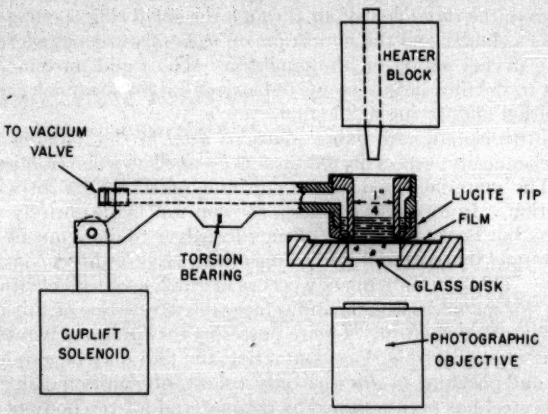

FIG. 2. Detail of processing cup.

Continuous cycling is controlled from a bank of switches operated by a corresponding bank of cams mounted on a single shaft. The shaft is driven through a one-turn clutch and completes one revolution in fifteen seconds. The cycle may be started at any desired instant but it is usually triggered at the completion of a single rotation of the radar antenna.

There are some unusual features involved in the film processing which contribute toward the extraordinarily quick production of a photographic image. This is made possible primarily by the rapid action of hot chemicals applied directly to the film surface. The application of the solutions to the film and the removal of these solutions after brief periods of chemical action is accomplished in a miniature "solution tank" (Fig. 2), consisting of two concentric cylinders with an

annular air space between them. The outer cylinder has at the bottom a flat polished lip that may be pressed firmly against the emulsion surface to flatten the film against the track bottom. The film thus becomes a bottom to the cylinder and forms a seal that will hold solution. At the side of the cup is a hollow arm that serves as a support and as a conduit for a suction line admitting to the air space between the inner and outer cylinders. The arm is mounted with a torsion bearing in such a manner that the cup may be either lifted clear of the film by the action of a solenoid or be pressed against the film by a spring when the solenoid is de-energized.

When a vacuum is applied to the suction line, any liquid contained in the cup is quickly removed through the hollow air space surrounding the inner cylinder, and if the suction is continued after the free liquid is removed, the rapid flow of air through the small ring opening between the inner cylinder and the film skims off the moisture and exerts a very effective drying action on the emulsion. The rapid movement of air tangent to the film surface seems to be essential for the quick removal of the solution and drying of the film.

Both developing and fixing solutions used in this process are very active chemically, especially because they are hot when applied to the film. The materials used in the cup unit must be able to withstand such action. In its original design, the cup was made entirely of stainless steel, but because of the tendency for silver to plate out of solution onto the steel this material proved unsatisfactory for direct contact with the film. This plating tendency occurs in much less degree with certain plastics, for instance, Lucite, and replaceable screw tips of this material have proved satisfactory. Lucite possesses the further advantage that a well-fitting polished surface contacting the film may be produced by lapping and polishing *in situ* with only a short interruption of operations. Stainless steel has been retained as the material for the body of the cup and has given no trouble.

Figure 3 is a schematic diagram of the solution feed system of the processing machine. Chemicals for the developing and fixing baths are stored in two constant-level supply tanks located above the light-tight enclosure. Feed lines run through a steel block, electrically heated and thermostatically controlled in temperature. The lines within the block contain about 2 cc. of solution each. Only a limited amount of liquid is heated, therefore, and it remains hot for only a short time before it is used. The tubes terminate in two nozzles located so as to direct their respective solutions into the cup.

Solenoid-operated needle valves at the supply tanks control the gravity flow of solutions. The amount of liquid that pours into the cup is determined by the duration of the solenoid signals. As usually set up, the valves are timed to be open for about 0.3 second, during which time about 0.3 cc. of liquid enters the cup.

The suction line is also controlled by a solenoid-operated valve. When this valve opens, the solution is sucked into a waste trap exhausted to a vacuum of about 20 inches of mercury.

The developer selected for use with this instrument is a modification of the Kodak Developer, D-82, in which about twice the normal quantity of caustic soda is used. For reasons to be given later, developer is

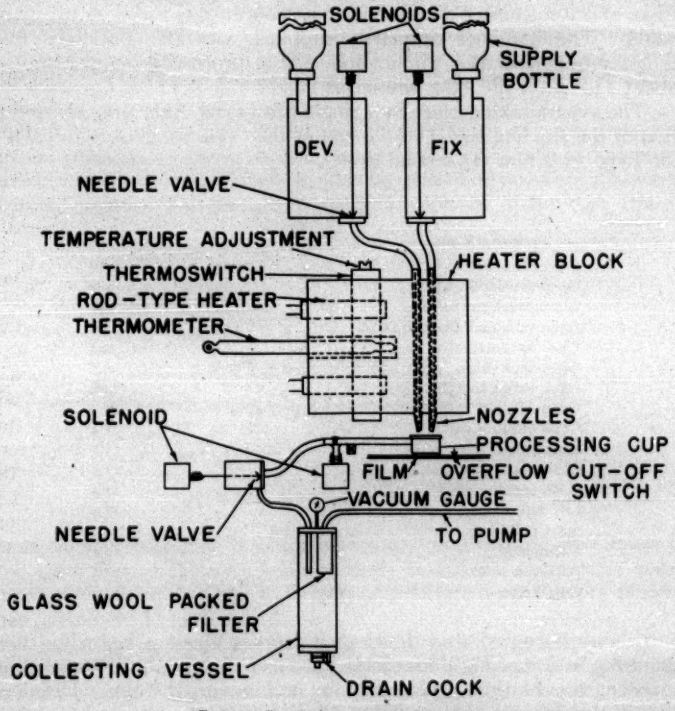

FIG. 3. Processing system of the P41.

used to rinse the film and the processing cup following fixation so that a humidifying agent—ethylene glycol—is added to prevent crystallization of the developer on the unwashed film after drying. The quantity of glycol required is about 10 per cent. of the volume of the mixed developer. The developer with these modifications may be used either full strength, or diluted up to 50 per cent. with water, depending on the temperature used and the rate of processing required. With some films

a small amount of antifoggant—benzotriazole— was added to maintain the background areas fog-free.

When the above-described full-strength developer is diluted, two parts developer to one of water, excellent contrast is obtained in 3 to 4 seconds' development with Eastman Fine Grain Release Positive Film, Type 5302, at a processing temperature of about 140° F.

The fixing bath used in the P[4]I recorder is Kodak Liquid X-Ray Fixer and Replenisher diluted, one part concentrated fixer to two parts water. The hardener ordinarily supplied with this packaged fixer is not required in this application. At a processing temperature of about 140° F., Type 5302 film clears sufficiently in about 6 seconds.

The events taking place in a single processing cycle from the reception of the starting signal to the end of the cycle are shown in Table I for Type 5302 film (15-second cycle).

Table I.

Sequence of Events	Time in Seconds
Start of camshaft	0.0
Open developer valve	0.1
Close developer valve	0.4
Open vacuum valve	4.0
Close vacuum valve	4.4
Open fixer valve	4.5
Close fixer valve	4.8
Open vacuum valve	11.0
Close vacuum valve	11.4
Reopen developer valve	11.5
Close developer valve	11.8
Open vacuum valve	12.5
Lift cup	13.9
Start pulldown	14.0
Vacuum off	14.1
Pulldown ends	14.3
Cup drops	14.4

It may be noted that development takes about 4 seconds, fixing about 6.5, and washing 1.0 second. The rest of the time is consumed in removing the solution, drying the film, and moving the film. Developing solution is used to rinse fixing solution from the film and from the cup because it is available without the addition of more plumbing. Unless the cup itself is rinsed free of fixing solution, the developer used for the next image will be contaminated and the resulting image will be fogged.

While images processed in this way have been kept under moderate conditions for a period of a year without apparent deterioration, it is not intended to imply that they would last for long periods, especially under unfavorable conditions.

Satisfactory projected images of reversal-processed film have been produced on the P⁴I equipment in less than 10 seconds. Briefly, this was accomplished by substituting a strong dichromate bleach, of the type capable of dissolving metallic silver (similar to the Kodak Tray Cleaner, TC-1) for the fixing bath. The complete processing cycle consisted merely of developing at about 135° F. for 6 seconds, and bleaching for 2 seconds. The resulting two-tone image then consisted of a semi-opaque silver halide background with clear images. The yellowish stain caused by the dichromate solution was unobjectionable for the present purpose. The contrast in the projected image between the clear areas and the background was found to be about 15 to 1. This was ample for the present application.

The image of the PPI screen (which may be of any size between 4.5 and 10 inches) is formed on the film as a circle 0.285 inch in diameter. In field radar operation, this circle must be imaged on an 8-foot-diameter plotting board. This entails a magnification of about 340 times.

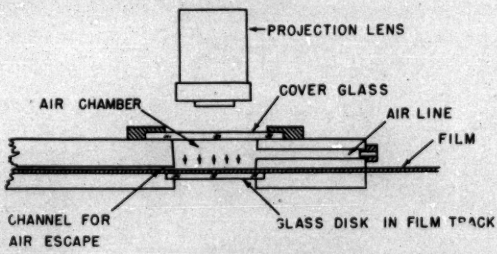

FIG. 4. Air pressure film gate.

Even though the screen brightness required for this projected image is low according to motion-picture theater standards, there were some special difficulties that had to be overcome before a satisfactory projection system could be attained.

The image-bearing film arrives at the projection gate in a slightly moist condition, since it has been dried for only a fraction of a second. It must remain in the projection gate for as long as 30 seconds. Screen definition over the entire area and for the entire time must be excellent and lateral distortion must be kept at a minimum.

A 400-watt lamp and a projection optical system of $f/3$ gave sufficient screen illumination and satisfactory definition under operating conditions, but the film was overheated in this system when it was left in the gate for the required time.

To overcome the tendency of the film to buckle and burn, heat-absorbing glass was inserted in the beam and a new type of air-cooled gate was devised. In this device, illustrated in Fig. 4, compressed air in an

enclosed chamber with a glass top maintains downward pressure on the film and holds it flat against the glass-bottomed film track. Air may escape from this chamber only in a direction tangent to the film surface —a kind of flow that facilitates both drying and flattening of the film. The effectiveness of the flattening effect of the air is evidenced by the appearance of Newton's rings over the entire screen as a result of the film's close contact with the plate glass. The presence of the interference pattern on the plotting board is not objectionable but the pattern may be made to disappear by a slight adjustment of the air pressure.

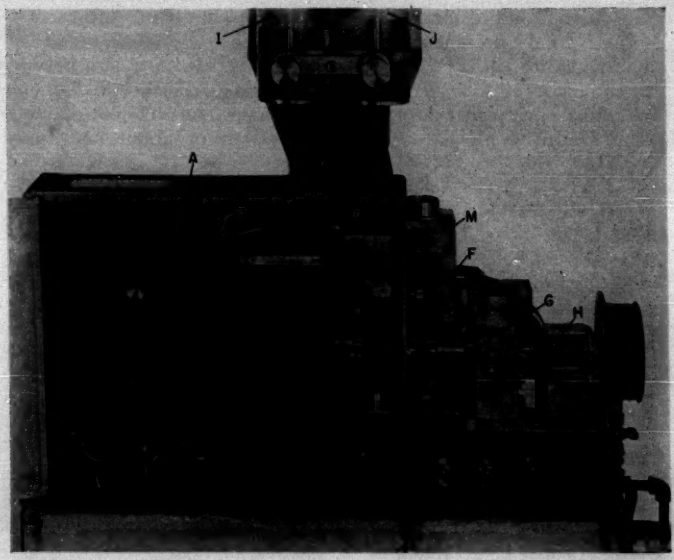

Fig. 5. Side view of P⁴I unit with camera door open. A, 400-ft 16-mm film magazine. B, unexposed film. C, film track. D, processing cup. E, air pressure projection gate. F, projection lens and prism. G, film pulldown. H, spot-processed film. I, developer. J, fixer. K, heater block. L, vacuum valve.

A small rotary pump serves the double purpose of providing the necessary cooling air from its outlet and supplying the vacuum for removal of solution from its inlet. Liquid waste is prevented from entering the pump by a large trap and a glass wool filter.

The film is indexed 1.5 inches by one turn of a steel roller against which the film is lightly pressed by a hard-rubber idler roller. The signal for the pulldown comes from one of the cam-operated switches.

This signal operates a solenoid that releases the knockout dog on a one-turn clutch. When in proper adjustment, this mechanism is capable of registration of successive images with a maximum jump less than $\frac{1}{8}$ inch on an 8-foot-diameter picture. This indicates that the film is positioned to within ± 0.0002 inch.

Many of the obvious mechanical and electrical control features

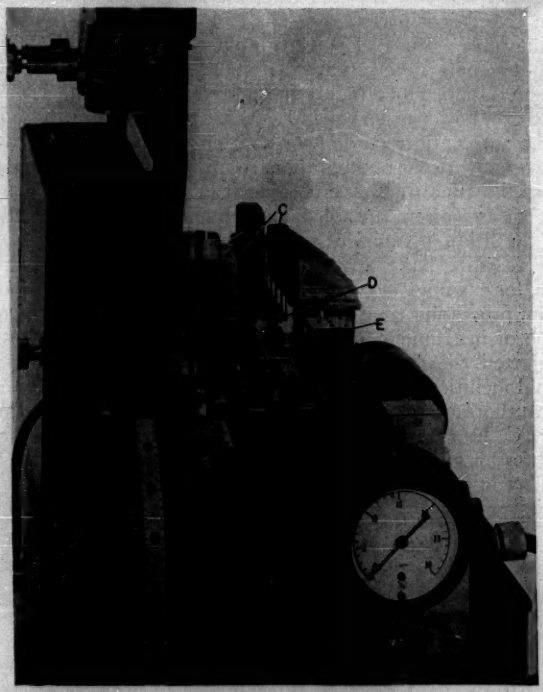

FIG. 6. End view of P4I unit on sub-base. A, projection lens. B, film pulldown. C, cams and microswitches controlling solenoids. D, camshaft clutch. E, camshaft-clutch releasing solenoid. F, pulldown clutch. G, pulldown solenoid. H, lamphouse.

necessary to satisfactory operation of the P4I equipment have not been mentioned in this discussion because they are of operational interest only. We should, however, like to include two pictures of the finished equipment. The first, Fig. 5, is a view of the unit with its door open. The more interesting parts have been labeled and no further description seems necessary. The second, Fig. 6, is a view of the equipment

mounted for field use upon a sub-base assembly which, in turn, is supported by a metal table. The PPI chassis is mounted on the other end of the table. Light from the tube face is turned upward into the photographic objective by a prism mounted on the sub-base. Within the sub-base is mounted all of the accessory equipment necessary to the operation and timing of the P4I. This equipment includes the sweep-timing Selsyns, the air pump, the lamp blower, etc. The waste solution chamber is attached at the end of the table. On a shelf beneath the table is the PPI power supply.

The foregoing description covers the instrumental aspects of an equipment designed for a highly specialized application. The principles involved are, however, so general as to be of interest in those processes requiring automatic recording or routine testing and inspection. In procedures now using photographic recording, such as in spectroscopic analyses, the time of film processing can be reduced to a matter of seconds in cases where suitable film is obtainable. In other operations, such as the study of transient phenomena, the rapidly processed photographic recording has many advantages. Comparison of two or more displays, for example, is greatly facilitated in a minimum of time. The short film-processing time and the completely automatic process make this type of instrument applicable to many similar procedures which have rejected the method of photography because of the time and additional facilities required.

The authors are happy to make acknowledgment to Mr. O. Wampole and Mr. C. E. Ives for some helpful suggestions, and to Mr. C. M. Tuttle * for his guidance in the capacity of co-ordinator of the work on this project in the Kodak Research Laboratories.

* Mr. Tuttle is a co-inventor on two pending patent applications dealing with the system.

NOTES FROM THE NATIONAL BUREAU OF STANDARDS.*

ABSORPTION MEASUREMENTS DEFINE CONCRETE WALL THICKNESSES FOR PROTECTION AGAINST BROAD BEAMS OF X-RAYS.

Concrete walls 51 inches thick will afford adequate protection against 2,000,000-volt broad-beam X-rays, and 36 inches thick against those of 1,000,000 volts, when the subject is 4 ft. from the source and the current is 3 milliamperes, according to data reported in the August *Journal of Research* (RP 1735). These are maximum thicknesses representing about the most severe conditions that may be encountered. At greater distances and with smaller currents, walls of less thickness will afford satisfactory protection.

The extensive industrial use of X-rays for inspection purposes necessitated precautions to protect the health of the large number of workers in this field; it is estimated that there was a 2,000-per cent. increase in personnel engaged in this type of work during the war. Although it has long been known that X-ray work can be carried out with complete safety when proper precautions are taken, it was necessary to establish detailed specifications for safeguards in this particular service. No data directly applicable to industrial conditions was available, as all previous efforts had been directed toward protection from narrow beams, that react in an entirely different manner from the broad beams used in industry. Accordingly a committee of the American Standards Association, with George Singer of the Bureau's X-ray Section as chairman, and representing industry, labor, hospitals, medical schools, insurance companies, and government agencies, undertook the development of a safety code for the industrial use of X-rays. This code has now been adopted as an American War Standard, Z54.1-1946.

Making use of the best biological information available, the committee adopted a permissible daily dose of 0.1 roentgen per day. In other words sufficient protection must be interposed between the source and a person so that he will receive no more than 0.1 roentgen in 24 hours.

As one phase of the program, absorption measurements were obtained for broad beams of 1- and 2-million volts X-rays, at the laboratories of the General Electric Company in Schenectady, by Carl B. Braestrup of the Department of Hospitals, New York City, and Harold O. Wyckoff and George Singer of the Bureau staff.

The observations were made in a room equipped with a clear opening between the X-ray source and the ionization chamber used for determin-

* Communicated by the Director.

ing the strength of the radiation. The opening, a doorway, was closed with concrete blocks that were built up in layers to make walls of different thicknesses. Suitable precautions were taken that there were no leaks around and between the blocks, so that the radiation measured was actually radiation which had passed only through the concrete.

The results are presented in curves and tables showing the thickness of concrete (density 2.35 g. per cu. cm. or 147 lb. per cu. ft.) required for primary protective barriers, for operation at different currents and at different target distances. A correction should be applied if the concrete is of a different density. Although described as preliminary results, they represent the best technical data as yet available, and are considered sufficiently reliable for the present purposes. As higher voltages are becoming available for use in generating X-rays, the work is being extended, with refinements in apparatus and procedure. At the same time the present data will be brought up to date, to reflect these improvements.

THEORY FOR AXIAL RIGIDITY OF STRUCTURAL MEMBERS HAVING OVALOID OR SQUARE PERFORATIONS.

In many modern structures, especially bridges, the columns have a built-up box-type cross section in which cover plates replace the conventional lattice bars and battens. The cover plates are perforated at intervals to allow access to the interior of the columns for inspection and painting.

The axial rigidity of a column is reduced by the presence of these perforations, that is, the column will shorten more for a given load than it would if the perforations were absent. A knowledge of the rigidity of the various members of a structure may be required for accurate analysis and design.

Formulas from which axial rigidity can be computed in the cases of circular, elliptical, and some types of ovaloid holes have been developed and published by the Bureau. This has now been extended to include cases in which the perforations are of various other shapes, including additional types of ovaloid as well as square perforations. Results of this work are reported by Martin Greenspan in the September *Journal of Research* (RP 1737).

RESINOUS SEALANTS FOR POROUS METAL CASTINGS.

The production of pressure tight castings is a foundry problem that frequently presents considerable difficulties. During the war the production schedules for certain fittings, housings, and other castings that had to be pressure tight, were in many instances hampered because of porosity rejections. It was suggested that many of these rejected castings might be salvaged by impregnation with suitable resins. Although

considerable work had been done in this field, very little useful information has been published.

Because of the widespread interest in this subject an investigation of the suitability of fourteen synthetic resins for repairing porous castings was carried out at the Bureau. This work is described in the September *Journal of Research* (RP 1740) by Vernon C. F. Holm of the Bureau's Division of Metallurgy.

Porous bushings of aluminum, a copper-silicon alloy, and a red brass were cast especially for this purpose in the Experimental Foundry, some to exact size, 2 × 2-in. cylinders with ⅛ in. walls, others over-size and machined to this size. Specimens were impregnated with the resins by the vacuum-pressure method, and the sealing efficiencies of the resins determined by hydrostatic pressure tests.

The two most promising sealants, a phenolic resin in alcohol solution and a styrene-polyester resin, were selected for further tests. Specimens sealed with these resins were exposed to hot motor oil, high octane gasoline, boiling water, elevated temperatures, and thermal shock. The influence of moderately high pressure on specimens treated with these two sealants also was determined. The styrene-polyester resin was more satisfactory for sealing castings that were exposed to petroleum products or subjected to thermal shock conditions. The phenolic resin was somewhat superior in exposure to elevated temperatures and specimens sealed with this resin withstood higher pressures without leaking. Both resins were reasonably stable in exposure to boiling water.

Although impregnation with sealants may be a useful method for rendering certain types of porous castings pressure tight for limited service conditions, the results of this investigation suggest that, in general, the foundryman should attempt to avoid porosity in castings rather than to cure it.

THE OPTICAL GLASS INDUSTRY, PAST AND PRESENT.

Although the history of optical glass dates from ancient times (magnifying lenses of that period have been unearthed), it was not until 1790 that its manufacture in the modern sense began. The invention of a means of stirring the molten glass by Pierre-Louis Guinand was the first major improvement in the industry. In 1880, the work of O. Schott at Jena, Germany, gave final impetus to the industry by making available many oxides not previously used.

The production of optical glass in the United States was started about 1893 by Macbeth and Co., Pittsburgh, Pa., with the assistance of Edmond Feil, a great-grandson of Pierre-Louis Guinand. This work has been continued to the present day, by the Bausch & Lomb Optical Co. (since 1912) and by the National Bureau of Standards (since 1914).

The various procedures used in the production of optical glass,

namely, pot making, melting, breaking open the pots and trimming, inspection, molding, and annealing are described briefly in a paper by Francis W. Glaze, prepared for publication in *Sky and Telescope*. The paper continues with a description of the methods used in producing two large telescopic disks in the United States: (1) the 70 inch disk for Ohio Wesleyan University by the National Bureau of Standards, and (2) the 200 inch disk for the California Institute of Technology by the Corning Glass Works. It is concluded that the optical glass industry in the United States is capable of fulfilling the requirements of optical designers and astronomers, no matter how fantastic these requirements may seem.

WAR-TIME ACTIVITIES ON PLATING AT THE NATIONAL BUREAU OF STANDARDS.

Emphasizing the important service that the electroplating industry rendered in the war, William Blum, Chief of the Electroplating Section, has outlined the Bureau's contributions in this field toward meeting specific new demands and making possible substitutions for strategic materials.

Among the more significant activities were those involving the protection of gun barrels from erosion and of steel cartridge cases against corrosion. As the result of a cooperative project between the Bureau of Standards and the Geophysical Laboratory, commercial production of chromium plated caliber-0.50 barrels was undertaken with War Department approval. Chromium plating was applied to the unlined portions of those barrels equipped with stellite breech liners, and also to barrels made from nitrided steel. These studies produced valuable information on the properties of electrodeposited coatings of the more common elements, such as chromium, zinc, and manganese, as well as of some alloys, for example, alloys of tungsten or molybdenum with iron, nickel, or cobalt.

Other problems arising from the scarcity of certain metals, notably copper, zinc, aluminum, chromium, nickel, and tin, received attention. For instance, a process was developed for producing iron electrotypes as a substitute for the common electrotypes of copper. The Bureau also cooperated with the War Production Board and other agencies in efforts to meet essential civilian needs with materials that were available.

Steel, when substituted for the non-ferrous metals, usually required protection against corrosion, which in many cases could be furnished satisfactorily by electroplating. Some of the materials protected in this way were zinc-coated steel one-cent coins, chromium-plated mess trays, and plated steel tableware. Experience with this latter item, particularly through actual tests in camp service, demonstrated the importance of these tests in evaluating coatings and in determining the

type of protection required. The tests also showed that plated coatings can be designed to meet very severe service conditions.

Special forms of the magnetic thickness gage ("Magne-gage") were designed and calibrated for measuring the thickness of very heavy coatings of nickel on steel, and the thickness of coatings on the inside of tubes such as gun barrels. As measurements with "Magne-gages" are non-destructive, these instruments were used extensively in testing coatings on many military supplies; over 2000 of these gages have been calibrated at the Bureau.

In this paper, which will be published in the Proceedings of the American Electroplaters Society, ways are suggested in which the knowledge gained through military applications may be applied to industrial processes. It is also pointed out that many of the investigations are being continued, and details will be published when the work is completed.

EFFECT OF REACTION BETWEEN MERCURY AND OXYGEN IN AQUEOUS SOLUTIONS UPON POLAROGRAPHIC WAVES OF CERTAIN METALS AT SMALL CONCENTRATIONS.

The polarographic method of analysis is useful for determining small amounts of metals in ores and alloys, plant ashes, biological materials, mineral waters, samples collected in industrial hygiene studies, and other materials. The presence of an unsuspected systematic error in such determinations might lead to expensive mistakes in the value of an ore or the composition of an alloy, or might cause unfortunate consequences through the approval of unsafe respiratory equipment. For example, if a polarographic analysis for lead in fumes were to give a negative result when lead was actually present, serious harm to the health of workers could result.

In the September *Journal of Research* (RP 1736), Edgar Reynolds Smith, John Keenan Taylor, and Roberta Evelyn Smith show that the polarographic waves of certain metals, notably of lead, zinc, copper, cobalt, and nickel in dilute neutral solutions, diminish progressively with time in the presence of mercury and air. This behavior is traced to a reaction between mercury, water, oxygen, and the ions of the metallic salt which, in some instances, precipitates the metal from solution in the form of its hydroxide. Recognition of this reaction and precautions to exclude it are essential for accurate polarographic determinations of small amounts of heavy metals.

Super-Speed X-Ray Exposures Study Body Distortions Fliers Endure.—Millionth-of-a-second X-ray pictures now are being used to study the bodily distortions a flier endures when he makes a crash landing or is catapulted out of a crippled speed plane, a Westinghouse scientist said recently.

Army and Navy aero medical laboratories have ordered X-ray units to obtain visual evidence of what happens to the pilot's spine, heart and other organs while his body is jolted by the sudden and terrific stops and starts involved, Dr. Charles M. Slack told the Connecticut Valley section of the American Industrial Radium and X-ray Society.

"In crash landings, for example," the scientist explained, "fliers undergo a shock equivalent to 10 times the shock to a motorist if he could halt his automobile in one second from an 85-mile-an-hour speed. Body harnesses have been devised to cushion such deceleration blows but the strain on organs still poses a problem."

By stimulating actual flying and crash conditions in the laboratory, X-ray studies of the body can be made while the body is subjected to strains, Dr. Slack pointed out. Sequence photographs can be arranged to show the step-by-step distortion of the organs, which are stopped dead by millionth-of-a-second exposures.

"Jet age planes are cracking old speed marks so fast that the Army Air Forces recently predicted that future planes would be zooming through the air at 1,000-mile-an-hour speeds, 300 miles faster than the speed of sound," Dr. Slack said. He added:

"If a man tried to climb out of a supersonic plane whose motor failed he would be cut to ribbons by the wind so a mechanical arrangement has been devised to literally fire him out of the airplane simply by pushing a button. The high accelerations necessary to get the pilot out so he can parachute to safety give his organs a jolt 20 times as severe as the normal pull of gravity."

X-ray study of ejection seat "body bending," as well as studies of the proper amount of explosive force required to fire pilots out of the cockpit, are included in the aero laboratory investigations.

The study of bodily distortions is the most recent of a number of unusual applications in the medical, ballistic and industrial fields for the X-ray unit using the tube, which was developed in the Westinghouse Research Laboratories here five years ago by Dr. Slack and his associates.

The unit was used during the war for experiments in connection with the atomic bomb as well as for studies of shells, bullets and armor plate. Peacetime possibilities include the study of swiftly-moving machine parts while they are moving.

The speedy exposures—20,000 times as fast as the normal exposure for routine medical X-rays—are accomplished because of a special electronic tube which operates under high momentray voltage. When this voltage is applied electrons speed at approximately 100,000 miles a second in their flight from the tube filament to the anode "target" at which the electrons are aimed, pouring an X-ray barrage through metal to make the picture.

R. H. O.

THE FRANKLIN INSTITUTE.

LIBRARY NOTES.

The Committee on Library desires to add to the collections any technical works that members would wish to contribute. Contributions will be gratefully acknowledged and placed in the library. Duplicates received will be transferred to other libraries as gifts of the donor.

Photostat Service. Photostat prints of any material in the collections can be supplied on request. Orders received in the morning are filled the same day. The average cost for a print 9 × 14 inches is thirty-five cents.

The library and reading room are open on Mondays, Tuesdays, Fridays and Saturdays from 9 A.M. until 5 P.M., Wednesdays and Thursdays from 2 P.M. until 10 P.M.

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NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION.

Alkaline Hydrolysis of and Effect of Heat on Ribonucleic Acid.—CHARLES A. ZITTLE. Variation in the ease of hydrolysis of different samples of ribonucleic acid with alkali has been observed (1, 2, 3). Since an aged sample of nucleic acid had become more difficult to hydrolyze (1), the effect of heat on nucleic acid was investigated as the most likely cause of the variations in hydrolyzability. The reactivity of the heated nucleic acids with the phosphodiesterase ribonucleinase and with a less specific phosphodiesterase (4) was studied as well. From these and other experiments in which the concentration of NaOH was varied it was concluded that the concentration of NaOH was critical for the hydrolysis of ribonucleic acid to mononucleotides. In addition to this stage of hydrolysis of the nucleic acid, measured by precipitation with the uranium reagent (decomposition of tetranucleotides), another stage was evident by the decrease in pH of the hydrolysates which continued when the former had ceased, and it, too, was characterized by a critical pH requirement.

PROCEDURE.

Commercial nucleic acids (free acid or sodium salt) were used as obtained or after purification by precipitation with 5 volumes of concentrated acetic acid (5). Crystalline ribonucleinase was prepared and its reactivity with nucleic acid determined by the manometric method (5, 6). The phosphodiesterase was prepared from calf intestinal mucosa and its activity determined manometrically (4).

In the early experiments neutral nucleic acid solutions (5 per cent.) were hydrolyzed by the addition of NaOH to give a concentration of 0.16 N. The concentrations of NaOH are, in every case, in excess of that required to neutralize the nucleic acid. In later experiments other concentrations of NaOH were used. The course of the hydrolysis was followed by precipitating the unhydrolyzed nucleic acid from 5.0 cc. of a 1 per cent. dilution of the nucleic acid with an equal volume of the uranium reagent (0.25 per cent. uranium chloride in 5.0 per cent. trichloroacetic acid) and the precipitate weighed (3). In a few cases the phosphorus soluble in this reagent was estimated and the results were parallel with those obtained by the above procedure. Changes in the pH during hydrolysis of the nucleic acid in the alkali were measured with a Beckman pH meter, laboratory model *G*, with the electrode for alkaline solutions. The meter was regularly checked against a $NaHCO_3$–Na_2CO_3 buffer of pH 10.85 (7).

RESULTS.

The course of hydrolysis of one sample of nucleic acid (sodium salt) with 0.16 N NaOH after various treatments is shown in Fig. 1. The sample when first obtained was completely hydrolyzed in less than 24 hours (curve *A*); after aging it was less readily hydrolyzed (curve *B*); after heating even less hydrolysis occurred (curve *C*). Another sample of nucleic acid of the same lot number when received gave the hydrolysis curve *D*; on heating at 65° for 16 hours it became more resistant

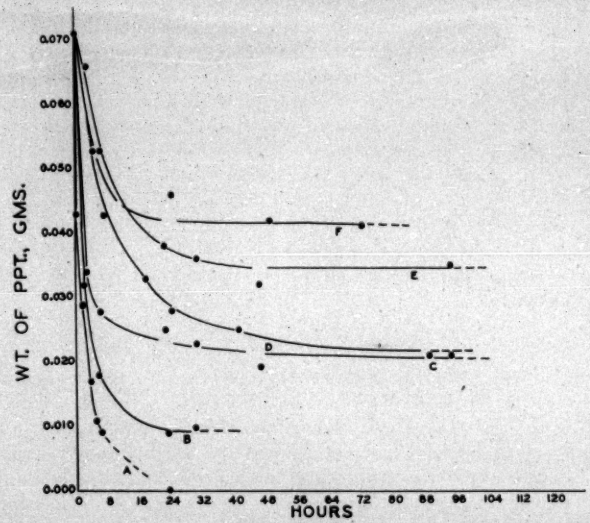

FIG. 1. Hydrolysis of yeast ribonucleic acid with 0.16 N NaOH: Eimer and Amend lot No. 410684. Sample 1: curve *A*, original; curve *B*, after 3 months at summer temperature; curve *C*, after heating 16 hrs. at 65°. Sample 2 (EA_{II}-2): curve *D*, original; curve *E*, after heating for 16 hrs. at 65°; curve *F*, after heating for 24 hrs. above 150°. The nucleic acids are in every case adjusted to the same concentration on the basis of the initial precipitate with the uranium reagent.

to hydrolysis as shown by curve *E*, which occurred to the same degree when it was heated to 90°, 115°, or 150° for 24 hours. At higher temperatures where pronounced charring occurred hydrolysis was somewhat less (curve *F*). Similar results were obtained whether the nucleic acid was heated as the sodium salt or the free acid. The sodium salt giving curve *B* was precipitated with concentrated acetic acid and subsequently with and without heating gave hydrolysis curves like *C* and *B*. The incomplete hydrolysis and the effect of heating was found also with commercial ribonucleic acids from several other sources.

In Table I are shown the dialyzability of the heated nucleic acids and their reactivity with ribonucleinase and a phosphodiesterase from calf intestinal mucosa (4). The effect of heat is noted first in the increased dialyzability of the nucleic acids and subsequently this change is reflected in decreased reactivity of the nucleic acids with the enzymes. The decrease in size of the heated nucleic acid molecules, demonstrated

TABLE I.

Comparison of Certain Properties of Heated and Unheated Ribonucleic Acids.

Sample	Treatment	Dialyzed*	Decreased reactivity with ribonucleinase†	Decreased reactivity with phosphodiesterase‡
		per cent.	*per cent.*	*per cent.*
M_I	Unheated	15.8	—	—
M_I	Heated 115°, 16 hrs.	28.6	0	0
M_I	Heated 170°, 16 hrs.	65.2	39.9§	
EA_{II}-2	Unheated	3.8	—	—
EA_{II}-2	Heated 300°, 1 hr.	36.6	0	0
EA_{II}-2	Heated 184°, 16 hrs.	43.2	31.7	0
E_I, purified	Unheated	18.3	—	—
E_I, purified	Heated 115°, 16 hrs.	32.5	0	0
E_I, purified	Heated 184°, 16 hrs.	60.7	81	24.6

* A rough measure of molecular size was obtained from the per cent. N dialyzing through a cellophane membrane under the following conditions: a 20.0 cc. portion of neutral 2.0 per cent nucleic acid solution was placed in a Visking 12/32 tube and suspended in 2.0 liters of water or buffer (0.0178 M sodium phosphate, 0.15 M NaCl, pH 7.0) at 7°. At the end of 48 hrs. the per cent. of N dialyzing was calculated from both the decrease of N inside the membrane and the N appearing outside the membrane.

† Determined in the region of adequate substrate concentration (4) with 100 mg. and 50 mg. of nucleic acid. The decreases were calculated from the 100 mg. determinations.

‡ Determined with phosphodiesterase (4) in the region of inadequate substrate concentration (5.0 and 2.5 mgms.) The decreases were calculated from the 5.0 mg. determinations.

§ Obtained with excess of substrate (200 mg.).

by the change in dialyzability, was accompanied by the appearance of acid groups measured by an increase in the titration with alkali.

The degree of hydrolysis with other concentrations of NaOH and the changes in the pH during the hydrolysis are shown in Figs. 2a and 2b.

DISCUSSION.

The hydrolysis of commercial samples of ribonucleic acid with 0.16 N NaOH ranges from complete (Fig. 1, curve *A*) to only about 50 per cent. A possible clue to this variation is the decrease in hydrolysis of one sample on aging (compare curves *A* and *B*, Fig. 1). Heat was thought most likely to have caused the change and the results show that the heated nucleic acid samples are less hydrolyzable.

The explanation for the dissimilarity in hydrolysis of the two sam-

ples of nucleic acid of the same lot number (curves *A* and *D*, Fig. 1) [1] is not apparent since neither was exposed to heat in storage but the latter may have been treated differently in the past. It is remarkable that the first sample remained readily hydrolyzable for more than four years [1] and then deteriorated on standing three months at summer temperature but it may not have been exposed to such temperatures previously.

Several of the nucleic acid samples that were poorly hydrolyzed were quite dark in color and it is probable that the dark color and incomplete hydrolysis were both caused by the same factor, i.e., excess heat at some

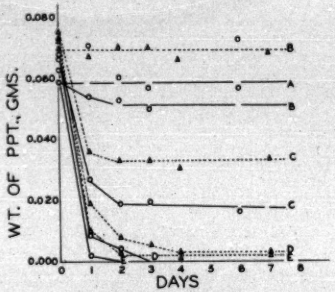

FIG. 2a. Hydrolysis of ribonucleic acid with various concentrations of NaOH. The final concentration of nucleic acid was 5 per cent. N NaOH was added to give the indicated concentrations. Samples were taken, precipitated with the uranium reagent and the precipitates weighed as before. Nucleic acid EA_{II}-2 (solid lines); NaOH, moles/liter: curve *A*, 0.04; curve *B*, 0.08; curve *C*, 0.16—the same as in previous experiments; curve *D*, 0.24; curve *E*, 0.32. Nucleic acid EA_{III}, purified (dotted lines); NaOH, moles/liter: curve *B*, 0.08; curve *C*, 0.16; curve *D*, 0.24; curve *E*, 0.32. The slight precipitate which was persistently obtained with the uranium reagent with the EA_{III} hydrolysates *D* and *E* probably was not nucleic acid since it was reddish brown in color whereas the initial precipitate with nucleic acid was white.

stage in their manufacture. Heating the nucleic acids has been found to darken their color as well as decrease their hydrolyzability.

Electrophoretic studies in our Physical Chemistry Department have shown that with 2 per cent. solutions of nucleic acid only traces of a component other than nucleic acid were observed and that the dark color accompanied the nucleic acid. Experiments with heated nucleic acids showed that there was no change in the mobility of the nucleic acid and that the increased color resulting from heating accompanied

[1] Obtained from Eimer and Amend, lot No. 410684. Dr. Harold A. Frediani, Director of Laboratories, for Eimer and Amend, states that this sodium nucleinate is of German manufacture and was received early in 1941. In storage it is adequately protected against excessive heat and light. These nucleic acids, in other than the extent of hydrolysis, are identical in every property studied except that the initial rate of hydrolysis of the first sample (curves *A*, *B*, and *C*) appears to be greater than that of the second sample (curves *D*, *E*, and *F*).

the nucleic acid. The color may represent an alteration of the ribose part of the molecule which could occur without affecting the charge on the molecule.

The enzyme data in Table I raised doubt whether fundamental changes in the structure of the nucleic acid were caused by heat since a change in reactivity with the enzymes was not detectable under conditions that had caused a decrease in hydrolysis with alkali. The appearance of acid groups on heating the nucleic acid suggested that the final concentration of alkali available for hydrolysis might be a factor.

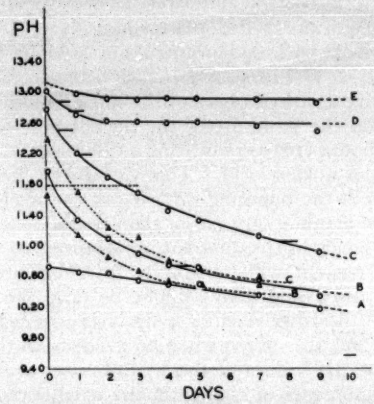

FIG. 2b. Change in pH of solutions of ribonucleic acid with various concentrations of NaOH. The conditions and the symbols are the same as for Fig. 2a. The relative buffering power of the nucleic acid (EA_{III}-2) solutions (before hydrolysis) at the different pH values was determined by adding one or several equivalent amounts of HCl to each. The horizontal lines show the pH values reached (that for *A* lies below the curve). The hydrolysates were kept stoppered to minimize the entrance of CO_2. Other buffers measured under the same conditions showed no decrease in pH.

Small, gradual changes in the pH had been noted during the course of hydrolysis with alkali but on account of the abrupt termination of hydrolysis (Fig. 1) little significance had been attached to them. Experiments with several concentrations of NaOH in which changes in pH were determined showed, however, that the pH was critical (Figs. 2a and 2b); when the pH fell below 11.8 (horizontal dotted line) hydrolysis stopped. Parallel results were obtained with the nucleic acids EA_{II}-2 and EA_{III}; the latter was less completely hydrolyzed than EA_{II}-2 with 0.16 N NaOH and the explanation for this is seen in the initial pH and the changes during hydrolysis.

The incomplete hydrolysis of the heated nucleic acids was also found to be due to a drop in the pH below 11.8. The nucleic acid giving curve

F, Fig. 1, with 0.16 N NaOH, for example, had at the end of 48 hours dropped below pH 11.8. The concentration of NaOH was increased to twice the initial concentration by the addition of 2.5 N NaOH and now hydrolysis was complete at the end of 24 hours.

Nucleic acid solutions slightly below the critical pH can be hydrolyzed on heating the solutions to higher temperatures; EA_{II}-2 with 0.16 N NaOH, for example, can be almost completely hydrolyzed by heating for 4 hours at 100°. Hydrolysis, however, proceeds much more easily and completely at 20–25° when sufficient alkali is added to give the required pH.

The significance of the critical pH for hydrolysis may lie in the fact that at 11.8 the ribose hydroxyl groups begin to ionize (pK_a' in nucleotides = 13.2–13.9; see ref. 10, p. 284). The original observations (1, 2) on the variable extent of hydrolysis of ribonucleic acid with alkali were thought to bear on the structure of the nucleic acid and recently the data have been quoted (10) as indicating a difference in the structure of different samples of nucleic acid. This variability, however, must be due to differences in the buffering capacity of the nucleic acids arising from differences in manufacture rather than to differences in the nature of the ribonucleic acids in the different samples.

The uranium reagent will precipitate tetranucleotides and so the hydrolysis under discussion above might be only the stage tetra- to mononucleotides. Another stage of hydrolysis appears evident from a study of Figs. 2a and 2b. Even when no decrease in the uranium precipitate occurs the pH values of the hydrolysates are decreasing and approach a minimum rate of change in the neighborhood of pH 10.6. Solutions of nucleic acid made alkaline to pH 10.6 show little decrease in pH. This stage of hydrolysis, probably polynucleotide to tetranycleotide, occurs rapidly only above pH 10.6. The changes observed cannot be due to differences in the buffering power of the solutions since solutions of nucleic acid at the initial pH values of curves *A* and *B*, for example, both have about the same buffering power (Fig. 2b). Bredereck and Hoepfner (8) have reported the preparation of tetranucleotides by alkaline hydrolysis of ribonucleic acid. Fletcher *et al.* (9) have been unable to duplicate their results but obtained a product of higher molecular weight. In view of the apparently critical pH requirement for this stage of hydrolysis (greater than pH 10.6, less than pH 11.8), it is evident that different nucleic acid samples might behave differently, depending on their buffer content.

SUMMARY.

The hydrolysis of ribonucleic acids with 0.16 N sodium hydroxide at 20–25°, measured by solubility in uranium reagent, differed greatly with different samples. All samples of nucleic acid were less readily hydrolyzed after heating in an oven at 65 to 184° for 16 hours. Heat

had little influence on the ability of the nucleic acids to serve as substrates for ribonucleinase or a phosphodiesterase from calf intestinal mucosa, although after heating at the higher temperatures there was a pronounced decrease in reactivity. The nucleic acids when heated were broken into smaller fragments with an increase in acid groups. Variations in the course of hydrolysis of the nucleic acids were explained by the finding that the pH for hydrolysis was critical; below a pH of 11.8 no hydrolysis to mononucleotides soluble in uranium reagent took place. The content of acidic substances in the nucleic acids, produced either in manufacture or by heating, accordingly determined the outcome of hydrolysis with the concentration of sodium hydroxide (0.16 N) first used. An earlier stage of hydrolysis than that measured by uranium reagent, measured by decrease in pH of the hydrolysates, also showed a critical pH requirement,—10.6.

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BOOK REVIEW.

DAVID RITTENHOUSE, ASTRONOMER-PATRIOT, 1732–1796, by Edward Ford. 226 p. port. 20 × 13 cm. Philadelphia, University of Pennsylvania Press, 1946. Price $2.50.

It is fitting that the 150th anniversary of the death of one of America's first scientists should be marked by the appearance of a new biography. Grandson of the first papermaker in the Colonies, Rittenhouse early acquired a taste for handicraft and in 1749 he built a shop where he began making clocks. He became interested in astronomy in 1756 and pursued his scientific studies on his own initiative. Among his most notable accomplishments were the construction of two orrerries or planetariums, and his observations of the transit of Venus in 1769 with his resultant calculations of the solar parallax.

Rittenhouse was interested in other sciences as well as astronomy for he advanced an hypothesis of the molecular theory of magnetism, although he has received little recognition for it. He used spider threads in transit instruments, and discovered the principle of the diffraction grating, whose value he foresaw in optical work, but Fraunhofer receives credit for the invention much later.

Mr. Ford reminds us that the scientific achievements of Rittenhouse might well have been greater if he had not been so involved in public affairs. He was engaged in surveys of the Pennsylvania-Virginia, the New York-Pennsylvania, and the New York-Massachusetts boundaries. He was an assemblyman and served on the Committee of Safety in 1776. From 1777 to 1789 Rittenhouse was state treasurer and during most of the period he was trustee of the Pennsylvania loan office, 1780 to 1790. He reentered public service a little later when Jefferson persuaded him to become the first director of the mint in 1792.

Active in the American Philosophical Society, Rittenhouse was honored by election as its president after the death of Franklin. Among other honors accorded him was election as a fellow by the Royal Society of London in 1795.

The present volume offers a new and modern interpretation of David Rittenhouse, his ife and his work. The author has maintained a rather even balance in his account of the scientific and public activities of Rittenhouse, but it is on his scientific achievements that the reputation of Rittenhouse rests, for his public service had little of the spectacular or momentous.

G. E. PETTENGILL.

PUBLICATIONS RECEIVED.

Le Cobalt, by Robert Perrault. 147 pages, illustrations, 16 × 26 cms. Paris, Dunod, 1946.

Forensic Chemistry and Scientific Criminal Investigation, by A. Lucas. 340 pages, 14 × 21 cms. London, Edward Arnold & Co., 1945. Price $7.00.

Endless Horizons, by Vannevar Bush. 182 pages, 15 × 23 cms. Washington, D. C., Public Affairs Press, 1946. Price $2.50.

Beams on Elastic Foundation, by M. Hetényi. 255 pages, drawings, 18 × 26 cms. Ann Arbor, The University of Michigan Press, 1946. Price $4.50.

The Metallurgy of Quality Steels, by Charles M. Parker. 248 pages, drawings and illustrations, 16 × 23 cms. New York, Reinhold Publishing Corporation, 1946. Price $6.00.

The Alkaline-Earth and Heavy-Metal Soaps, by Stanley B. Elliott. 342 pages, illustrations, 15 × 23 cms. New York, Reinhold Publishing Corporation, 1946. Price $7.50.

CURRENT TOPICS.

One of World's Largest Dragline Excavators.—A giant electric dragline excavator, one of the largest in the world, is now working at the Maumee Collieries Corp., Terre Haute, Indiana. Equipped with General Electric motors and control equipment and constructed by Bucyrus-Erie, the dragline has a bucket capacity of 25 cubic yards, equivalent to 37 tons of earth and rock overburden. It weighs close to 2,400,000 pounds, and is capable of moving a million cubic yards of dirt per month, which would uncover 75,000 tons of coal for smaller electric shovels to scoop up.

The total equivalent horsepower rating of the electric equipment installed on this giant machine is well over 6,000 hp. Two motor-generator sets, each driven by a synchronous motor, convert the incoming alternating current to suitable variable-voltage direct current for operating the various motions of the dragline. Amplidyne control provides smooth and flexible control of all motions.

The machine can make a complete cycle of operation on a short swing in slightly over 30 seconds and a long swing of approximately 180 degress in 50 to 60 seconds. Material can be picked up at one point and deposited at another point fully 350 feet away. The machine can dig to a depth of 100 feet.

A unique walking arrangement permits the dragline to pull itself along on extremely soft terrain. When the machine is picking up material it rests on a large central circular platform or "tub." When it must be moved to a new location, however, immense boats or shoes at the sides are lowered and, acting like a pair of crutches, raise the machine off the tub and slide it forward to a new position where it again rests on the tub. This operation is repeated until the machine reaches its destination.

R. H. O.

Hydraulic Speed Gears.—The application of hydraulic speed gears as power drives was discussed in a paper by E. R. Boynton of General Electric's Aeronautics & Marine Engineering Divisions at the recent semi-annual meeting of the American Society of Mechanical Engineers in Detroit. A hydraulic speed gear, Mr. Boynton stated, consists essentially of two hydraulic units, one bieng driven and supplying a flow of oil under pressure to a second unit which drives the load. The purpose of the speed gear is usually to drive a load at varying and controlled speeds from an essentially constant-speed source of power or one whose speed is not easily controllable.

These gears are especially suitable for aircraft applications, Mr. Boynton continued, since they have very low weight per horsepower. Because of the characteristically low inertia of the rotating parts of a hydraulic motor, it is especially attractive for control jobs requiring rapid accelerations and decelerations. Other advantages of hydraulic speed gears are their ability to maintain torque at standstill without heating, and ability to limit torque easily and accurately.

Stating that the average speed gear leaves something to be desired from the user's point of view, Mr. Boynton said gears of the future should have longer life, greater simplicity and lower cost, higher efficiencies and speeds, should be quieter in operation, and that more emphasis should be placed on the engineering and integration of complete systems.

R. H. O.

Uses for Rutin Expanding.—Clinical experience with the drug, rutin, is expanding its range of usefulness in treatment of diseases associated with failing blood vessels. Apoplexy, some forms of persistent nose-bleed, retinal hemorrhage, inflammation of the eyes often associated with diabetes, and other diseases in which a tendency to hemorrhage is an important symptom, have been treated with success in addition to the initial use of the drug in cases of increased capillary fragility.

Dr. J. Q. Griffith, Jr., of the Medical School of the University of Pennsylvania has been cooperating with Dr. J. F. Couch of the Eastern Regional Research Laboratory of Philadelphia where the experimental work was done that has developed a commercial method of producing rutin. Dr. Griffith, handling the clinical phase of the work, reports that rutin has been supplied to many physicians and hospitals and that results have been encouraging. In the conditions of increased capillary fragility, Dr. Griffith says the conditions of the capillaries have been brought to normal in 85 per cent. of the cases.

Dr. Griffith says that apoplexy has been a disease terrifying to victims and their families because a first "stroke" was so likely to be followed by another and more serious one. Treatment with rutin has proved beneficial in a large percentage of the cases treated, by restoring the capillaries and thus preventing a recurrence of the attack. This applies, also, to cases of retinal hemorrhage, likely to cause blindness, and to retinitis or inflammation of the retina often associated with diabetes. Griffith says that various investigators report that reduction of blood pressure, tonic effects on the heart, and diuresis are the principal effects of rutin treatments. Cases of persistent nose-bleed are corrected because rutin treatment strengthens the capillaries from which the bleeding comes.

The drug has been known as a chemical substance for more than a century. Its medical values were not detected until U. S. Department of Agriculture research workers extracted it from flue-cured tobacco and noted its chemical similarity to "vitamin P." When clinical tests of rutin derived from tobacco proved promising, the researchers turned to other possible sources of the drug, and found that young buckwheat plants are a much richer and cheaper source of rutin. Several drug manufacturers are now processing buckwheat to supply rutin, and Dr. P. A. Wells, Director of the Eastern Laboratory, estimates that there may be an annual demand for growing up to 50,000 acres of buckwheat to supply the new drug.

R. H. O.

Liquid Assets Concentrated.—"Liquid assets" of the American people —that is, the sums that individuals could spend in the near future if they decided to do so—are rather closely concentrated in a relatively small proportion of people. This is the conclusion announced by the U. S. Department of

Agriculture as the result of a survey early this year which the Board of Governors of the Federal Reserve System asked the Bureau of Agricultural Economics to make.

A quarter of the families have no liquid assets. This group lives from current income. At the other end of the scale, 10 per cent. of the spending units have 60 per cent. of the liquid assets, and the upper quarter hold nine tenths of the liquid assets, the half in the middle about one-tenth.

The term "liquid assets" includes: (1) government bonds (particularly war savings bonds); and (2) bank deposits (savings and checking accounts). The survey talks in terms of the "spending unit," defined as "all persons in a household who depend on a common or pooled income for their major expenses." Earlier the bureau made a pilot survey by personally questioning a small carefully selected sample. Results were so significant that the Reserve officials requested this survey as a guide to what may be expected in the postwar economy.

The announcement of results of the survey by the bureau emphasizes that because of the concentration of so large a proportion of the liquid assets in a relatively small part of the population, the effect of the liquid assets "will depend in the main on how a relatively small part of the population decide to use their holdings." The bulk of spending is expected to come from the current income of all spending units, as is always the case.

"People's expressed intentions for 1946 indicate," says a summary, "that several billions of liquid assets will be used for consumption and investment during the year. But, just as before the end of the war, most people consider their liquid assets as earmarked for long-range purposes and not available for current expenditures; therefore, they intend to finance most of their planned expenditures, including those for durable goods and houses, out of current income or by borrowing.

"According to people's present expectations, they will save considerably less in 1946 than they did in 1945, even if incomes are good. Those who accounted for most of the 1945 savings expect to save much less this year. Some of them plan to spend income for large items not previously available, and some feel that higher prices will compel them to spend more for living expenses."

R. H. O.

Plastic Bearings Aid in Search for Greater Power.—Unique experiments with transparent plastic models of machine bearings are revealing "inside information" that may result in more powerful motors for the nation's factories, research engineers of the Westinghouse Electric Corporation disclosed recently.

The plastic models, which glisten with the brilliance of costume jewelry, are enabling scientists to look inside a moving bearing and see what is happening to the lubricating oil—something never before possible, according to John Boyd, Westinghouse research engineer who developed the novel technique.

"One peek at these plastic models in operation often is better than weeks of mathematical calculations," Mr. Boyd explained. "Guess-work is eliminated. Now we can see what is happening and can design bearings which will handle a heavier load and help make possible more powerful motors without any increase in size or weight.

"Our procedure is simple. Oil, colored with a red pigment, is fed into the

bearing model and the lubrication effectiveness quickly is apparent. We know at once whether the seals work properly, whether the oil passages are enabling a proper flow of the lubricant, and whether the relief points are located correctly. All this formerly had to be determined very largely by trial and error."

Blocks of Lucite, a transparent plastic, are machined to close tolerances just like metal at the Westinghouse Research Laboratories to form the models of bearings and journals. During the actual operating tests, the rotating element of the finished model is operated by a hand crank for slow speeds and is motor-driven for high speeds.

As the red-colored oil works its way through the bearing, engineers can see whether the grooves which distribute the oil are properly placed to do a thorough job. If these oil passages are not in the right position, some areas will be "starved" for oil and the bearing cannot carry a fully load. This, in turn, reduces the amount of work which the motor can handle.

After watching the transparent models in operation, the engineers can redesign the bearing so that thorough lubrication is assured. This enables the entire length of the bearing to shoulder the burden and the motor can do a bigger job.

"The design of machine bearings in the past has been governed by certain mathematical equations based on 'ideal' conditions," Mr. Boyd pointed out. "How well the flow of lubricant in an actual bearing approximated these conditions remained a mystery until this test method was developed.

"Such questions as the proper position from which to feed lubricants into bearings, the right depth and angle of oil grooves, and the pressure necessary to force grease and oil into a bearing all lacked definite answers.

"Now we are getting these answers and can take one more step in the constant march toward getting more and more power from smaller and smaller motors."

R. H. O.

Oils from Grasses.—One of the "war baby" industries of Latin America is the production in Guatemala of the grass oils—citronella and lemongrass oils. The products are used in soap and perfumery, for scenting insecticides, and for other technical purposes. The grass oil growers pioneered in the decade before the war and found tough going, according to Graham S. Quate of the U. S. Department of Agriculture, reporting to the Office of Foreign Agricultural Relations.

The decade of trade depression in the 30's made for small profits and discouragement. With competition reduced by war, prices were good, acreage was expanded, and the grass specialists have returned profits to improving both the growing and the distilling branches. They hope to be able to continue as producers of these oils.

Citronella and lemongrass are related plants and the practices of production and the products are similar. In a warm climate and with good moisture—either from rain or from irrigation—these perennial grasses sprout promptly after harvest and are ready for cutting every 90 days, making the growing and distilling year round operations. After about three years (12 cuttings) a field deteriorates and needs a rest or the planting of a soil building crop. Growers keep down weed competition because weeds in the harvested grass would in-

jure the quality of the distilled oil. The crop is cut by hand, allowed to wilt for a day, and is then hauled to the steam distilling plant for extraction of the oil.

R. H. O.

Mums at Any Season.—The chrysanthemum is the football flower and the Thanksgiving standby because it is so regular in its fall blooming habits. But by taking advantage of this habit that is as regular as a football schedule, the florist can offer chrysanthemums for St. Valentine's Day—if customers favor the idea—or for Christmas or New Year decorations.

More than many plants, the chrysanthemums of a given variety are regular in blooming at a given date, because the blooming habit is controlled by the length of day. When Drs. Garner and Allard of the U. S. Department of Agriculture were making their studies a generation ago of what is known as photoperiodism in plants, they soon identified the chrysanthemum as a plant that formed flower buds as the days shortened. By moving a chrysanthemum into a dark house each afternoon and so shortening the day artificially, they found they could force a plant to bloom weeks ahead of a sister plant left in natural sunshine. Florists were able to take advantage of this discovery and could force early blooming by covering a bench of chrysanthemums with a tent of dark cloth each day— an artificial sunset.

Continuing similar studies, Neil W. Stuart of the Bureau of Plant Industry, Soils, and Agricultural Engineering has found a way to get the opposite effect. He finds it is possible to delay the formation of buds in chrysanthemums by growing them in a greenhouse and breaking the long night by expoisng the plants to electric lighting. This then has the effect of two short nights instead of one long one, and budding is prevented or delayed. For simplicity and practical result an hour of light at midnight seemed most desirable in the early experiments. But similar delays in blooming followed short exposures several times during the night.

The timing experiments were interrupted by the war, but have been resumed. It will be necessary for experimenters or florists to work out lighting schedules to suit early and late varieties of the flower. By moving potted flowers into and from the midnight-lighted greenhouse at successive dates and allowing the buds to form normally, it will be practical to keep different pots of a single variety blooming for two months or more of the winter. The same effect, a succession of bloom from a single variety, will be obtained if the plants are not moved, but instead use of the lights is discontinued successively along the bench.

Lighting should be started before the buds have formed, otherwise the quality and number of the flowers may be reduced. Use of lights in winter and dark cloth in summer will permit the florist to grow only the best varieties and have them in bloom throughout the year.

R. H. O.

Lead Makes Invisible Infrared Rays Visible.—Invisible infrared rays are made visible by the presence of lead in a material which glows after exposure to light, Dr. Gorton R. Fonda, of the General Electric Research Laboratory has found. This may help in equipment with which to see in the dark.

Waves of infrared radiation are too long to affect the eye, while those of

ultraviolet are too short. However, the latter may easily be made visible by the phenomenon of fluorescence, used in the fluorescent lamp. Here ultraviolet rays are generated inside the tube, fall on the "phosphor" with which the tube is lined, and there are changed into visible light.

Since changes of this kind must always be from shorter to longer waves, fluorescence by itself cannot make infrared waves visible, it could only make them still longer. However, there is an indirect way in which they can make light come from a phosphor.

Some of these materials show phosphorescence, as the emission of light continues for a time after the original radiation has been removed. If, while the glow remains, the phosphor is exposed to infrared, the brightness may be very slightly increased. After that it quickly fades out.

Dr. Fonda's research was concerned with a different type of excitation, reported originally by Franz Urbach, of the University of Rochester. Even after the original glow has faded completely, an exposure to infrared causes emission of light. Dr. Fonda has found that the effect occurs with zinc sulfide, a common phosphor, provided it contains a fraction of a per cent. of lead.

With ordinary fluorescence, electrons in the atoms of which the phosphor is made are knocked out of their normal orbits and into higher states by ultraviolet rays. As they fall back to their usual position, fluorescence occurs. With phosphorescence, the electrons are delayed in their return. According to Dr. Fonda, in the kind of action he has been studying the electrons are "trapped" in a higher state, from which they are released only by infrared rays, provided the lead "key" is present. The light given off is green in color.

With the "sniperscope" and "snooperscope" developed during the war, vision in total darkness was possible. The subject was illuminated with infrared, and a special electron tube combined with a telescope made the scene visible. Perhaps similar effects could be obtained with the infrared-sensitive phosphors.

Dr. Fonda's work, done during the war under the auspices of the Optics Division of the National Defense Research Committee, is reported in a paper in the Journal of the Optical Society of America.

R. H. O.

Forest Fires.—Merchantable timber, young growth not yet merchantable and acres of seedlings, valued in all at more than $5,000,000 were among timber resource losses that went up in the smoke of forest fires during the year 1945. Total timber and property damage from forest fires, on both protected and unprotected land, during the year was estimated at $26,726,919.

This was made known with the publication of the U. S. Department of Agriculture's Forest Service report on forest fire statistics for the calendar year 1945 gathered by that agency's field offices, 42 cooperating States, the Soil Conservation Service, the Department of Interior and the Tennessee Valley Authority.

The forest fire report showed that despite the importance of the lost timber to the national veterans' housing program and to other users already affected by the timber shortage, approximately 27 per cent. of the 1945 forest fires on protected areas were reported as of incendiary origin. No estimate as to causes of

fires on unprotected land are obtainable because of the lack of protection forces there, but incendiarism is believed to be also relatively high in these areas.

The deliberate setting of forest fires was the largest single cause of forest fires during the year, the report said. As given, the "cause record" for 1945 read: Incendiarists, 15,376; smokers, 12,569; debris burners, 7,630; miscellaneous, 7,376; lightning, 6,713; railroads, 4,307; campers, 1,864; and lumbering, 936. During the year, 4,157 persons were prosecuted for violations of State fire laws relating to forest fires on State and private lands alone, and convictions were obtained in 3,803 cases. On the same lands, 18 persons lost their lives while fighting forest fires.

During the year there was a total of 124,728 fires as compared with the reported total for 1944 of 131,229, or 6501 fewer. Of the 1945 fires, 45 per cent. were on the 516,152,000 acres of land under protection, while 55 per cent. took place on the 126,959,000 acres still without it. Despite the decrease in the number of fires reported, both acreage burned over and total damage to timber and other property showed small increases. Acerage burned in 1945 on protected and unprotected areas was figured at 17,680,703 as against 16,549,312 for 1944. Total property damage on both protected and unprotected areas in 1945 was estimated at $26,726,919 as compared with $25,775,312 in 1944.

On protected lands alone, total damage was reported as $8,611,839, of which $5,264,154 was to timber and the remainder to non-timber property. The reported money damage to commerical timber and property improvements represents only a small part of the ultimate loss in watershed protection and future merchantable timber.

The eleven Southern States which make up the Forest Service Southern Region showed the largest figures for acreage burned over, due to the fact that less than one-half of the southern forests have thus far been put under organized State protection. In "the South," 1945 saw a total of 15,341,268 acres burned. The figures for the other regions were: Pacific, 909,799; North Central, 725,041; Eastern States, 413,880; Rocky Mountain, 290,715.

Listed in the report, but not included in the figures for continental United States, were the 1945 forest fires statistics for protected public and private lands of Alaska. These lands suffered 90 fires as against 79 in 1944, and the acreage burned was 118,015, as compared with 110,585. Only one Alaska fire was reported as incendiary, the main causes being lightning 30, railroads 15, campers 13, and debris burners 11.

R. H. O.

Quick Test For Quinine.—In control of malaria, quinine from the Cinchona tree remains a standard medicine. Cinchona growing is now staging a comeback as a crop in several Latin-American nations and plant breeders and growers are interested in developing and growing trees whose bark supplies a high percentage of quinine. Seedling trees vary widely, and it would be a help in either breeding or planting to be able to know while a tree is young whether it can qualify as a heavy producer of quinine.

Working at the Federal Experiment Station in Puerto Rico, Arnaud J. Loustalot and Caleb Pagan have devised a test for quinine that takes only a fraction of the time needed for other tests, making it practical to test large

groups of seedlings and to discard any with a low yield of quinine. The new test depends on use of the laboratory device known as the spectrophotometer and calls for only a 2-gram sample of bark instead of the 10 to 20 gram samples required in older tests. This makes practical the testing of young plants that would be killed by removal of the larger sample. The new test, says the U. S. Department of Agriculture, has proved almost as accurate as the older and slower tests and can be used commerically in assaying the quinine content of Cinchona bark.

R. H. O.

Atomic Electric Generating Plants.—The time when atomic energy can be put to work on the big peacetime job of producing electric power at "slightly less" cost than in present day coal-fired plants is "much closer at hand than most people think," two scientists of the Westinghouse Electric Corporation reported recently.

The two men, Dr. J. A. Hutcheson, Associate Director of the Westinghouse Research Laboratories, and C. F. Wagner, Manager of the Central Station Engineering Department, disclosed that a survey of the relative costs of electric power produced by atomic energy and that produced in coal-fired plants has been prepared for the Company under the direction of Mr. Wagner.

"This survey," they reported, "indicates that the possible application of atomic energy to the production of electric power is sufficiently feasible as to warrant a careful and thorough investigation. From what we can see now it appears that technical problems, rather than economic problems, are the ones which must be solved before the atomic power plant of the future is practical."

The report says:

"An assumption is made that an atomic powered 100,000 kilowatt plant would be built, in which the cost of the equipment and plant necessary to provide steam for the turbines would be about $12,000,000. This is roughly four times the cost of the steam end of an equivalent power plant using coal as a fuel.

"Calculations were made comparing the cost of power obtained from this atomic power plant with that obtained from a coal power plant. These calculations included amortization of the investment in each case at the rate of 15 per cent. per year. It was further assumed that the atomic fuel would be refined natural uranium such as was used in the 'piles' at Hanford and Oak Ridge (atomic bomb manufacturing sites). Assuming that this material costs $20 a pound, the total cost of the generation of electric power in the atomic power plant appears to be slightly less than is the case in the coal-fired plant, assuming that coal costs $5 a ton."

The scientists explained that a plant with a capacity of 100,000 kilowatts could supply the electrical requirements of a community of approximately 300,000 persons.

At present, they added, there is continuous production of large quantities of controlled atomic power in the form of heat at both Hanford and Oak Ridge but it is a by-product and is carried away in one plant by air and in the other by a stream of water. By inference, they said, it seems reasonable to guess that this power must amount to many thousands of kilowatts. Therefore it seems logical to suspect that the earliest applications of atomic power will be similar to the present scheme of obtaining power from coal, in that steam will

probably be used to drive the turbine and that the heat liberated in connection with the fission of some material will be used to form the steam.

"If this is so, the problems associated with this work will be those brought about by the replacement of the present boiler and coal combustion equipment with some suitable heat exchanger and atomic power equipment," the survey stated.

The chief difficulty in the way of an atomic power plant is the need for special shielding equipment to confine the "very intense and dangerous radiations" emanating from the uranium fuel, the report said, adding that steel or concrete several feet thick would probably be needed.

"These radiations induce artificial radio activity in most things they strike," it continued. "This means that the steam passing through the turbine for example will probably be radioactive. The by-products of fission also are radioactive. Presumably these products will need to be removed from time to time. Consequently, special methods of handling and storing them will have to be provided.

"These problems are new to the electrical industry and much work will have to be done before it will be possible for a practical power plant to be built utilizing this new form of energy."

R. H. O.

Target Range Miner.—(*Compressed Air Magazine*, Vol. 51, No. 7.) When Uncle Sam tore Johnny Ballance away from his mining job in the Southwest and put him in a G.I. uniform, he started an interesting chain of events that launched Johnny in a strange sort of mining business. Assigned to the Air Corps, Ballance spent some time at a gunnery school outside Las Vegas, Nev., where the embryo flyers peppered bird shot at clay pigeons to train them to hit moving targets. He figured that quite a lot of lead was being shot into the desert, and in the course of some spare-time prospecting determined that the pellets didn't penetrate much beyond the surface of the ground. When his war service was over, Johnny asked for a lease on the former shooting range. The Government said he would have to compete for it, and asked for public bids. Several mining companies sent experts in to look things over, but they submitted discouraging reports, estimating that there was no more than a ton of lead to be recovered.

So Johnny got the contract and started work with a crew of ex-G.I's. With shovels, they loaded the top layer of sand into trucks and hauled it to a home-made separator that Johnny had rigged up. An integral part of this mechanism was a compressed-air jet that blew away the sand but left the heavier shot. There were also some screens and a water line, but the final separation was accomplished with air supplied by a small gasoline-engine-driven compressor.

Johnny's computations proved to have been better than those of the mining experts, for the desert gave up an even 200 tons of lead. This was sold at six cents a pound. The Government took a royalty of half a cent, and 1½ cents went to pay the crew. Ballance's share was four cents, or an aggregate of $16,000. At present he's looking over the rifle, revolver, and machine-gun ranges of the country and may be applying for another lease any day now.

Engineers in a New Role.—(*Compressed Air Magazine*, Vol. 51, No. 7.) Having demonstrated that there is practically no limit to their ability to build or rebuild inanimate structures, engineers are now turning their talents to reconstituting the human body. The appalling loss of limbs by participants in the recent war has served to emphasize the previous lack of serious study of the problem of supplying amputees with effective prosthetic devices. In an effort to correct this regrettable oversight, the Surgeon General of the U. S. Army has asked the National Research Council to set up a committee on prosthetic devices. This has been done, and its membership consists of three orthopedic surgeons and three engineers.

It is believed that engineers can help to bring about better artificial limbs by developing materials that are strong, light, waterproof, and easily formed; by devising simple methods of fastening the parts together; by designing an artificial knee that will bear load while flexed and a hand that can be manipulated to give it greater usefulness; and by contributing fundamental knowledge of biomechanics and determining the most essential motions and associated forces.

One of the subjects under consideration is the possibility of supplying artificial hands with some source of power to manipulate them in simulation of natural motions and functions. One of the suggested means of doing this is the use of small cylinders of compressed gas such as are employed to recharge soda-water syphons. Calculations have indicated, however, that with the valves now available the gas would be expended so wastefully that the wearer could not carry an adequate supply.

The conventional material of which artificial limbs have been made is wood, usually willow, covered with rawhide. Plastics have been adopted more recently, and in 1912, when Marcel Desoutter lost a leg in an airplane accident in England, he and his brother devised a substitute fabricated of duralumin. Magnesium has not been acceptable to date because it presents a corrosion problem that has not been solved.

One of the things the committee is working on is the standardization of artificial limbs to bring down the manufacturing cost. As a starting point, measurements have been taken of thousands of persons in an effort to determine a few standard sizes that would be suitable in most cases. In this connection it is noted that, in addition to the thousands of amputees among service men who must be supplied with prosthetic devices, there are between 25,000 and 40,000 amputations in an average year among the civilian population of the country.

In attacking this human engineering problem, technologists are citing that engineering is frequently defined as the application of the forces and materials of nature for the benefit of mankind. Clearly, this definition embraces the worthy cause in which they are now engaged.

R. H. O.

Electrical Experiments in Germany.—(*Electrical Engineering*, Vol. 65, No. 7.) According to a recent report published by the British Intelligence Objectives Subcommittee, 550 yards of experimental 400-kv a-c line had been constructed in order to study corona loss in connection with a proposed 480-mile transmission of 600,000 kva from the Alps to Cologne. Tests showed

that an arrangement of four conductors in parallel for each phase with 10-inch spacing between the steel-cored aluminum conductors would provide the maximum current-carrying capacity with the minimum of corona loss. Single-phase transformers with a 400-kv bushing out of one end and a 220-kv bushing for connection to a Petersen coil at the other, with lower voltage bushings at both ends, were to have been used. The air-blast circuit-breakers to be installed, rated at 1,000 amperes and 6,500 megavolt-amperes interrupting capacity, had 14 breaks in series with capacitors for promoting equal voltage distribution between breaks.

Experiments also were performed with d-c transmission with the ultimate idea of obtaining power from Norway and further supplies from the Alps. Tests were successful enough to warrant planning transmission of 160,000 kw by means of two 400-kv d-c lines from a station on the Elbe to the Marienfeld station in Berlin, 100 miles away.

R. H. O.

Electron Tube Converts Mechanical Motion Directly.—(*Electrical Engineering*, Vol. 65, No. 7.) Development of a tiny metal electron tube, the Vibroton, which converts mechanical motion directly into variable electron flow has been announced by the Radio Corporation of America laboratories at Princeton and Harrison, N. J. The tube weighs less than an ounce and is only about one inch in length and one-quarter of an inch in diameter.

Though the new tube is not yet in production, the company will make a limited number available to manufacturers of electronic equipment who are interested in experimenting with it for use in future products. It is expected that the tube will find wide application in future designs of phonograph pick-ups. Other fields of application are in microphones and in industrial equipment where translation of mechanical motion to electron circuits is desirable for purposes of control or measurement.

The tube is a metal triode, at one end of which is an extremely thin flexible metal diaphragm through which external motion is transferred to a movable electrode within the tube. Leads for supplying voltages are brought out through the other end. The tube operates as an integral part of the pickup head and the radio phonograph amplifier without the need of a preamplifier or coupling transformer.

Life tests have shown the tube to withstand severe treatment over long periods and to be stable under temperature and humidity changes.

R. H. O.

Shellac as a Moulding Plastic (*Journal of Industrial Research*, June, 1946). —Shellac is one of the oldest plastics known to mankind and took its place in industry shortly after hard rubber and bitumen mouldings, excelling them in every respect. As the base for gramophone records it has not been approached even to this day by any synthetic resin. Moulded shellac caskets were made in England in 1876. Shellac compositions for plastics were patented in 1868. Shellac gramophone records were perfected during the period 1890 to 1910. The next decade saw the development and use of shellac mouldings, in the electrical industry with such a wide variety of fillers like asbestos, mica, Kaolin, paper pulp, etc. Synthetic resins have not been able to replace shellac

to any considerable extent in gramophone records, micanite, laminated insulations, abrasive wheels. The greatest need for improvement in shellac plastics lines in increasing the heat-resistance and alkali-resistance and conferring thermo-hardening properties. Shellac although a thermoplastic like cellulose acetate and acrylic and styrol resins, does not possess their toughness for injection moulding, but the addition of fibrous fillers improves it to a certain extent for the manufacture of switches, small containers, etc.

W. A. R. P.

Natural Gas in the United States (*The Engineer*, Sept. 6, 1946).—Natural gas represents 12 per cent. of the power development in the United States as compared with 51 per cent. coal and 28 per cent. petroleum. It is used by 40,000,000 customers in 34 states. Industrial uses account for 78 per cent. of the consumption. Reserves are considered adequate for increasing expansion and several new pipe lines for transmission are being planned. Some pipe lines built originally for oil are being converted for the passage of natural gas. At the present rate of consumption the proved reserves would be sufficient for thirty-five years, but in view of prospective developments it is considered that these reserves will probably suffice for nearly 100 years. Natural gas has been used successfully as fuel for automobiles, but this use is not likely to be extensive. As a fuel, its ease of control, uniformity and freedom from ash are of primary importance, but it is important also as a source of carbon black. The gas as it reaches the consumer has had removed from it most of the high-boiling hydrocarbon components and averages 82 to 95 per cent. methane.

W. A. R. P.

Motors to Restore German Industry (*Technology Review*, July, 1946).—German industry has received a staggering blow and is completely disorganized. Today amid scenes of great destruction, life and business in Germany are being reorganized to a strange and uncertain future. Cities which were spared destruction are few and far between but there are a considerable number of manufacturing plants which have survived the war with little more than a single bomb hit. Machine tools have come through the bombings surprisingly well. The same cannot be said, however of the electric motor industry, which bears a large burden in the industrial rebuilding of Germany. Many of the electric motor plants are severely damaged. In some plants temporary repairs to buildings have made it possible to resume operations. The military government is sending hundreds of motors to be rewound, reconditioned or even completely rebuilt. Motors for street railways, electric locomotives, sanitation plants, refrigeration plants, are among those for which repair or replacement is most urgently needed. In Germany, the size and weight of a motor reflect the general practice of trying to make a little go a long way. Before the war, sheer necessity forced Germany to build motors with a very small available reserve for overload. The Germans had a lot of aluminum and used it instead of copper for the stator winding of squirrel cage motors. Some motors were also built using aluminum for commutators. The aluminum commutators were not considered satisfactory because of the rapidity with which they roughen from arcing. Better results were obtained with soft iron commutators

with thin sheets of copper between the iron and the insulation. Whether Germany is to rise again as an industrial nation will not be settled for some time to come. Meanwhile the German electric motor industry has its hands full in simply rehabilitating equipment damaged or virtually destroyed.

W. A. R. P.

Development of Efficient High-temperature Marine Gas Turbines (*Shipbuilder and Marine Engine-Builder*, Sept., 1946).—Since John Barker patented the first gas turbine in 1791, engineers and fuel technologists have expended much effort on its development into a practical and efficient prime mover, but until recently, progress has been slow. Under the stimulus of world war, progress in the development and application of the gas turbine as the power unit for aircraft has been rapid and successful, and today, the jet propulsion system, powered by gas-turbine plant is employed in the world's fastest aircraft. Progress in the adaption of the gas turbine to the industrial field (marine propulsion, locomotive power and electricity generation) has been less spectacular, but in the United States about 40 power plants of up to 6000 KW. are employing full or partial gas-turbine circuits; while a gas turbine locomotive has been completed and tested in Switzerland. In connection with marine propulsion, a gas turbine constructed at the Annapolis Station for the United States Navy, has been successfully operated at 1350 deg. F. At present a temperature in the neighborhood of 1000 deg. F. is a safe operating figure with the materials available. The search for improved materials continues, and it is expected that gas temperatures of about 1500 deg. F. at the turbine inlet will shortly be practicable, and a much higher thermal efficiency is anticipated. Among the main advantages of gas turbines for industrial power are the all-rotary motion of the units, the absence of water, simplicity of service and the ability to utilize cheap, low-grade fuels.

W. A. R. P.

New All-Steel Electric Motor Adds 'Drive' to Production Lines.—A new all steel-encased electric motor, reduced in size by as much as one-third but packing up to 134 per cent. more power per pound of weight, has been developed to help increase the efficiency of machine tools and put more "drive" into the nation's production lines.

This motor represents the most revolutionary change in construction since the invention of the electric motor 58 years ago. It is the first truly all-steel motor made available to industry.

High efficiencies and power factors have been maintained; shock resistance is increased many fold; smooth, quiet operation is improved; and maintenance requirements are materially decreased.

In the 7½ horsepower line the turning power when the motor is started has been increased 134 per cent. per pound of motor weight, while the maximum power has been increased 116 per cent. on the same basis. This has been accomplished without the sacrifice of electrical life or efficiency.

The frame for the new line is made from a single piece of steel rolled into a circle with the ends welded together. Obviously this frame is stronger than one made of cast iron. The same goes for the bell-shaped covers for each end

of the motor. In laboratory tests the motor stood up under repeated heavy hammer blows. It was undamaged in drops from the bed of a truck to the platform.

Motors of this type—from five to 75 horsepower—are urgently needed in the automobile, petroleum and chemical industries and by machine tool manufacturers.

Keynote of the new motor is its "flexibility," with the design so worked out that maximum use is made of a minimum number of parts.

Focal point of the flexibility built into the new motor centers upon the stator core (circular stationary part) and the frame assembly. The identical stator core and frame assembly is used regardless of whether the finished motor will be drip-proof, splash-proof, totally enclosed fan-cooled, or totally enclosed non-ventilated. As a comparison, the old line motor required a different frame casting for each of these types of enclosure.

Construction of vertical or horizontal motors without feet represents another phase. To obtain the round, footless frame all that is required is to leave off the steel foot assembly. In conventional type frames, this means a different frame casting. Then, too, the new motor was designed so that the bearing housings in each type of bracket have the same location. This necessitates the use of only one set of shaft dimensions and enables all shafts for a given size motor to be made in the same grinding set-up.

W. A. R. P.

District Heating System of Paris (*Industrial Heating Engineer*, July, 1946). —District heating has made enormous progress in the world during recent years. The chief country is, of course, the United States with some 170 plants. Canada, Soviet Russia, Belgium, Czecho-Slovakia, Denmark, France, Holland and Switzerland have made rapid strides in this field. Great Britain however does not yet posses a single installation of any size or importance. One of the most interesting district heating plants in the world is that of Paris, burning a considerable amount of town refuse and coal. The district heating system is operated by a private company formed in 1927 and now operates under direct control of the Municipality of Paris. Preliminary work was carried out in 1928 and heating steam was commenced in 1929 using for the purpose the old Bercy power station. Results were very successful and after 1932 the system was rapidly extended.

In 1934 to 1938 the total production of steam was about 55,000 tons per annum. During the war and after July 1940 scarcity of coal caused numerous difficulties. The heating system stopped in March 1944 and was not resumed during 1944–45. During the 1945–46 season the consumption of coal had been reduced to 50 per cent., but the company received a certain tonnage of fuel oil and combustible household refuse was also used.

W. A. R. P.